Australian Meteorological and Oceanographic Journal 64 (2014) 37–66

Paradigms for tropical cyclone intensification

Michael T. Montgomery[1] and Roger K. Smith[2]
[1]Department of Meteorology, Naval Postgraduate School, Monterey, CA
[2]Meteorological Institute, University of Munich, Munich, Germany.

(Manuscript received January 2013; revised March 2013)

We review the four main paradigms of tropical cyclone intensification that have emerged over the past five decades, discussing the relationship between them and highlighting their strengths and weaknesses. A major focus is on a new paradigm articulated in a series of recent papers using observations and high-resolution, three-dimensional, numerical model simulations. Unlike the three previous paradigms, all of which assumed axial symmetry, the new one recognises the presence of localised, buoyant, rotating deep convection that grows in the rotation-rich environment of the incipient storm, thereby greatly amplifying the local vorticity. It exhibits also a degree of randomness that has implications for the predictability of local asymmetric features of the developing vortex. While surface moisture fluxes are required for intensification, the postulated 'evaporation-wind' feedback process that forms the basis of an earlier paradigm is not. Differences between spin up in three-dimensional and axisymmetric numerical models are discussed also.

In all paradigms, the tangential winds above the boundary layer are amplified by the convectively-induced inflow in the lower troposphere in conjunction with the approximate material conservation of absolute angular momentum. This process acts also to broaden the outer circulation. Azimuthally-averaged fields from high-resolution model simulations have highlighted a second mechanism for amplifying the mean tangential winds. This mechanism, which is coupled to the first via boundary-layer dynamics, involves the convergence of absolute angular momentum within the boundary layer, where this quantity is not materially conserved, but where air parcels are displaced much further radially inwards than air parcels above the boundary layer. It explains why the maximum tangential winds occur in the boundary layer and accounts for the generation of supergradient wind speeds there. The boundary layer spin-up mechanism is not unique to tropical cyclones. It appears to be a feature of other rapidly-rotating atmospheric vortices such as tornadoes, waterspouts and dust devils and is manifest as a type of axisymmetric vortex breakdown. The mechanism for spin up above the boundary layer can be captured approximately by balance dynamics, while the boundary layer spin-up mechanism cannot. The spin-up process, as well as the structure of the mature vortex, are sensitive to the boundary-layer parameterisation used in the model.

Introduction

The problem of tropical cyclone intensification continues to challenge both weather forecasters and researchers. Unlike the case of vortices in homogeneous fluids, the tropical cyclone problem as a whole, and the intensification problem in particular, are difficult because of their convective nature and the interaction of moist convection with the larger scale circulation (e.g. Marks and Shay 1998).

There have been considerable advances in computer technology over the past several decades making it possible to simulate tropical cyclones with high resolution numerical models having a horizontal grid spacing as small as approximately 1 km. Nevertheless, important questions remain about their fluid dynamics and thermodynamics in addition to the obvious practical challenges to successfully forecast the intensity changes of these deadly storms

Corresponding author address: Prof. Michael T. Montgomery, Naval Postgraduate School, Monterey, CA 93943, USA. E-mail: mtmontgo@nps.edu

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **2014**	2. REPORT TYPE	3. DATES COVERED **00-00-2014 to 00-00-2014**
4. TITLE AND SUBTITLE **Paradigms for Tropical Cyclone Intensification**		5a. CONTRACT NUMBER
		5b. GRANT NUMBER
		5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)		5d. PROJECT NUMBER
		5e. TASK NUMBER
		5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Naval Postgraduate School,Department of Meteorology,Monterey,CA,93943**		8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSOR/MONITOR'S ACRONYM(S)
		11. SPONSOR/MONITOR'S REPORT NUMBER(S)
12. DISTRIBUTION/AVAILABILITY STATEMENT **Approved for public release; distribution unlimited**		
13. SUPPLEMENTARY NOTES		
14. ABSTRACT		
15. SUBJECT TERMS		

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**30**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

(e.g. Davis et al. 2008). Further evaluation of the intensity forecasts produced by high-resolution weather prediction models is certainly necessary in order to develop an appreciation of the strengths and weaknesses of the models in comparison with observations. It is unlikely, however, that significant advances will be made on the intensity-change problem without the concurrent development of a suitable theoretical framework that incorporates the dominant fluid dynamical and thermodynamical mechanisms. Such a framework is necessary to permit a more complete diagnosis of the behaviour of the model forecasts and to identify the essential processes that require further understanding and improved representation. It may be that a paradigm shift is required to break the intensity forecast deadlock.

Although one of the major impediments to intensity forecasts is believed to be associated with the interaction of a tropical cyclone with the vertical shear of the ambient wind (e.g. Riemer et al. 2010, Tang and Emanuel 2010 and references), it is imperative to have a firm understanding of the physical processes of intensity change for hypothetical storms in environments with no background flow. Such storms are the focus of this paper.

Over the years, several theories have been proposed to explain the intensification of tropical cyclones, each enjoying considerable popularity during its time. The three most established theories are based on axisymmetric considerations and include: Conditional Instability of the Second Kind (CISK); Ooyama's cooperative intensification theory; and Emanuel's air-sea interaction theory (or WISHE[1]). A fourth theory that has emerged from analyses of more recent cloud-representing numerical model simulations (Nguyen et al. 2008, Montgomery et al. 2009, Smith et al. 2009, Fang and Zhang 2011, Nguyen et al. 2011, Gopalakrishnan et al. 2011, Bao et al. 2012, Persing et al. 2013) highlights the intrinsically non-axisymmetric nature of the spin-up process and suggests a modified view of the axisymmetric aspects of the intensification process.

In the light of current efforts in many parts of the world to improve forecasts of hurricane intensity, especially cases of rapid intensification near coastal communities and marine assets, we believe it is timely to review the foregoing theories, where possible emphasising their common features as well as exposing their strengths and weaknesses[2]. An additional motivation for this review was our desire to interpret recent data collected as part of the Tropical Cyclone Structure 2008 (TCS08) field experiment (Elsberry and Harr 2008). While our main focus centres on problems related to the short-range (i.e. 4–5 day) evolution of storms, a more complete understanding of the mechanisms of tropical cyclone spin-up would seem to be useful also for an assessment of climate-change issues connected with tropical cyclones.

To fix ideas, much of our discussion is focussed on understanding various aspects of the prototype problem for tropical cyclone intensification, which examines the evolution of a prescribed, initially cloud free, axisymmetric, baroclinic vortex in a quiescent environment over a warm ocean on an f-plane. For simplicity, and for didactic reasons, the effects of an ambient flow, including those of ambient vertical shear, are not considered. Omitting the complicating effects of an ambient flow and vertical shear permits a clear comparison between the new paradigm and the old paradigms, which were historically formulated and interpreted for the case of a quiescent environment on an f-plane.

It is presumed that the initial vortex has become established and has maximum swirling winds near the ocean surface as a result of some genesis process. This problem has been studied by a large number of researchers. The aim of the paper is not to provide an exhaustive review of all the findings from these studies, but rather to review the various paradigms for tropical cyclone spin-up and to provide an integrated view of the key dynamical and thermodynamical processes involved in the spin-up process. The paper is aimed at young scientists who are just entering the field as well as those more established researchers who would like an update on the subject. In particular, our essay presents a more comprehensive view of the intensification process to that contained in the recent World Meteorological Organization sponsored review by Kepert (2011), which until now was the most recent word on this subject.

The paper is structured as follows. First, in 'Basic concepts of vortex dynamics and spin-up', we review some dynamical concepts that are required for the subsequent discussion of the various paradigms for tropical cyclone intensification. These concepts include: the primary force balances in vortices; the thermal wind equation; the meridional (or overturning) circulation as described by balance dynamics; the role of convergence of absolute angular momentum in the spin-up process; and the role of latent heat release and the frictional boundary layer in generating low-level convergence. We decided to include this section for those who are relatively new to the field and it is there that we introduce most of the notation used. Some of our readers will be familiar with much of this material and, since the notation is mostly standard, they may wish to jump straight to 'The CISK-paradigm'. In this and in 'The cooperative-intensification paradigm' and 'The WISHE intensification paradigm' we discuss the three most established paradigms for intensification. Then, in 'A new rotating-convective updraft paradigm' we articulate a new paradigm for intensification, in which both non-axisymmetric and azimuthally-averaged processes play important roles. Azimuthally-averaged aspects of this paradigm are examined in 'An axisymmetric view of the new paradigm'. Unbalanced aspects of axisymmetric spin-up are considered in 'Two spin-up mechanisms' and balanced aspects are considered

[1]The term WISHE, which stands for wind-induced surface heat exchange, was first coined by Yano and Emanuel (1991) to denote the source of fluctuations in subcloud-layer entropy arising from fluctuations in surface wind speed.

[2]Although other reviews have appeared during the past seven years (e.g. Emanuel 2003, Wang and Wu 2004, Houze 2010, Kepert 2011), the review and results presented here are complementary by focusing on both the dynamical and thermodynamical aspects of the intensification process.

in ‘An axisymmetric balance view of spin-up’. ‘Comparison of spin-up in axisymmetric and three-dimensional models’ examines briefly the essential differences between spin-up in three-dimensional and axisymmetric models. ‘Further properties of the rotating convective updraft paradigm’ examines properties of the new paradigm for the case of a quiescent environment on an *f*-plane. There, we describe tests of the dependence of the spin-up process on the postulated WISHE feedback mechanism and on the boundary layer parameterisation. Following ‘Conclusions’, our view of future directions is given in ‘The road ahead’.

Basic concepts of vortex dynamics and spin-up

We commence by reviewing some basic dynamical aspects of tropical cyclone vortices and key processes germane to their spin-up. For simplicity, we focus our attention first on *axisymmetric balance* dynamics and discuss then the departures from balance that arise in the frictional boundary layer.

The primary force balances

Except for small-scale motions, including gravity waves and convection, a scale analysis of the equations of motion for a rotating stratified fluid (Willoughby 1979) shows that the macro motions within a tropical cyclone are in a state of close hydrostatic equilibrium in which the upward-directed vertical pressure gradient force per unit mass is balanced by the gravitational force acting downwards

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = -g, \qquad ...(1)$$

where p is the (total) pressure, ρ is the moist air density, z is the height above the ocean surface, and g is the acceleration due to gravity. The scale analysis shows also that if the azimuthal mean tangential wind component squared is much larger than the corresponding radial component squared and if frictional forces can be neglected, a state of gradient wind balance prevails in the radial direction wherein the radial pressure gradient force per unit mass is balanced by the sum of the Coriolis and centrifugal forces

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{v^2}{r} + fv, \qquad ...(2)$$

where r is radius from the axis of swirling motion, v is the tangential velocity component, and f is the Coriolis parameter ($2\Omega \sin \varphi$, where Ω is the earth’s rotation rate and φ is the latitude).

Throughout this paper we take f to be constant on the assumption that the latitudinal extent of air motions within the vortex circulation is sufficiently small to render corresponding variations of f negligible: this is the so-called *f*-plane approximation.[3]

The validity of gradient wind balance in the lower to middle troposphere in tropical cyclones, above the frictional boundary layer, is supported by aircraft measurements (Willoughby 1990a, Bell and Montgomery 2008), but there is some ambiguity from numerical models. In a high-resolution (6-km horizontal grid) simulation of Hurricane *Andrew* (1992), Zhang et al. (2001) showed that the azimuthally-averaged tangential winds above the boundary layer satisfy gradient wind balance to within a relative error of 10 per cent, the main regions of imbalance being in the eyewall and, of course, in the boundary layer (see also, Persing and Montgomery 2003, Appendix A). A similar finding was reported by Smith et al. (2009) and Bryan and Rotunno (2009). However, in a simulation of Hurricane *Opal* (1995) using the Geophysical Fluid Dynamics Laboratory hurricane prediction model, Möller and Shapiro (2002) found unbalanced flow extending far outside the eyewall region in the upper tropospheric outflow layer.

In the next few sections we will focus primarily on the macro (non-turbulent) motions within a tropical cyclone vortex and parameterise the convective and sub-grid scale motions in terms of macro (or coarse-grained) variables. Then, in terms of the macro variables, the tropical cyclone consists of a horizontal quasi-axisymmetric circulation on which is superposed a thermally-direct transverse (overturning) circulation. These are sometimes referred to as the ‘primary’ and ‘secondary’ circulations, respectively. The former refers to the tangential or swirling flow rotating about the central axis, and the latter to the transverse or ‘in-up-and-out circulation’ (low- and middle-level inflow, upper-level outflow, respectively). When these two components are combined, a picture emerges in which air parcels spiral inwards, upwards and outwards. The combined spiralling circulation is called energetically ‘direct’ because the rising branch of the secondary circulation near the centre is warmer than the sub-siding branch, which occurs at large radial distances (radii of a few hundred kilometres). When warm air rises (or cold air sinks), potential energy is released (Holton 2004, p339). As a tropical cyclone becomes intense, a central cloud-free ‘eye’ forms, or at least a region free of deep cloud. The eye is a region of subsidence and the circulation in it is ‘indirect’ i.e. warm air is sinking (Smith 1980, Shapiro and Willoughby 1982, Schubert et al. 2007).

Thermal wind

Eliminating the pressure in Eqns. (1) and (2) by cross-differentiation gives the so-called ‘thermal wind equation’:

$$g\frac{\partial \ln \rho}{\partial r} + C\frac{\partial \ln \rho}{\partial z} = -\frac{\partial C}{\partial z}, \qquad ...(3)$$

[3]The *f*-plane approximation is defensible when studying the basic physics of tropical cyclone intensification (Nguyen et al. 2008, section 3.2.1), but not, of course, for tropical cyclone motion (e.g. Chan and Williams 1987, Fiorino and Elsberry 1989, Smith et al. 1990).

which relates the radial and vertical density gradients to the vertical derivative of the tangential wind component. Here

$$C = \frac{v^2}{r} + fv \qquad \text{...(4)}$$

denotes the sum of the centrifugal and Coriolis forces per unit mass (see Smith et al. 2005). Eqn. (3) is a linear first-order partial differential equation for $\ln\rho$. The characteristics of the partial differential equation satisfy

$$\frac{dz}{dr} = \frac{C}{g}. \qquad \text{...(5)}$$

and the density variation along a characteristic is governed by the equation

$$\frac{d}{dr}\ln\rho = -\frac{1}{g}\frac{\partial C}{\partial z}. \qquad \text{...(6)}$$

The characteristics coincide with the isobaric surfaces because a small displacement (dr, dz) along an isobaric surface satisfies $(\partial p/\partial r)dr + (\partial p/\partial z)dz = 0$. Then, using the equations for hydrostatic balance and gradient wind balance ($\partial p/\partial r = C\rho$) gives the equation for the characteristics.

The vector pressure gradient force per unit mass, $(1/\rho)(\partial p/\partial r,\ 0,\ \partial p/\partial z)$ equals $(C, 0, -g)$, which naturally defines the 'generalised gravitational vector', $\mathbf{g}_e$, i.e. the isobars are normal to this vector. Given the environmental vertical density profile, $\rho_a(z)$, Eqns. (5) and (6) can be integrated inwards along the isobars to obtain the balanced axisymmetric density and pressure distributions (Smith 2006, 2007). In particular, Eqn. (5) gives the height of the isobaric surface that has the value $p_a(z)$, say, at radius R.

The overturning circulation

Where the thermal wind equation is satisfied, it imposes a strong constraint on the evolution of a vortex that is being forced by processes such as diabatic heating or friction. Acting alone, these processes would drive the flow away from thermal wind balance, which the scale-analysis dictates. In order for the vortex to remain in balance, a transverse, or secondary circulation is required to oppose the effects of forcing. The streamfunction of this overturning circulation can be obtained by solving a diagnostic equation, commonly referred to as the Sawyer-Eliassen (SE) balance equation, which we derive below. This equation provides a basis for the development of a theory for the evolution of a rapidly-rotating vortex that is undergoing slow[4] forcing by heat and (azimuthal) momentum sources (see 'A balance theory for spin-up' later in this section). It is convenient to define $\chi = 1/\theta$, where θ is the potential temperature. (Note that, on an isobaric surface, χ is directly proportional to the density). Then, the thermal wind Eqn. (3) becomes

$$g\frac{\partial\chi}{\partial r} + \frac{\partial(\chi C)}{\partial z} = 0\,. \qquad \text{...(7)}$$

[4] Slow enough so as not to excite large-amplitude unbalanced inertia-gravity wave motions.

The tangential momentum and thermodynamic equations take the forms

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} + fu = F_\lambda\,, \qquad \text{...(8)}$$

and

$$\frac{\partial\chi}{\partial t} + u\frac{\partial\chi}{\partial r} + w\frac{\partial\chi}{\partial z} = -\chi^2 Q\,, \qquad \text{...(9)}$$

respectively, where u and w are the radial and vertical velocity components, t is the time, F_λ is the tangential component of the azimuthally-averaged force per unit mass (including surface friction), and $Q = d\theta/dt$ is the diabatic heating rate for the azimuthally-averaged potential temperature. Neglecting the time derivative of density, which eliminates sound waves from the equations, the continuity equation takes the form

$$\frac{\partial}{\partial r}(\rho r u) + \frac{\partial}{\partial z}(\rho r w) = 0\,. \qquad \text{...(10)}$$

This equation implies the existence of a scalar streamfunction for the overturning circulation, ψ, satisfying

$$u = -\frac{1}{r\rho}\frac{\partial\psi}{\partial z}, \qquad w = \frac{1}{r\rho}\frac{\partial\psi}{\partial r}\,. \qquad \text{...(11)}$$

The SE-equation for ψ is obtained by taking $\partial/\partial t$ of Eqn. (7), eliminating the time derivatives that arise using Eqns. (8) and (9), and substituting for u and w from (11).[5] It has the form

$$\begin{gathered}\frac{\partial}{\partial r}\left[-g\frac{\partial\chi}{\partial z}\frac{1}{\rho r}\frac{\partial\psi}{\partial r} - \frac{\partial}{\partial z}(\chi C)\frac{1}{\rho r}\frac{\partial\psi}{\partial z}\right] + \\ \frac{\partial}{\partial z}\left[\left(\chi\xi(\zeta+f) + C\frac{\partial\chi}{\partial r}\right)\frac{1}{\rho r}\frac{\partial\psi}{\partial z} - \frac{\partial}{\partial z}(\chi C)\frac{1}{\rho r}\frac{\partial\psi}{\partial r}\right] = \\ g\frac{\partial}{\partial r}\left(\chi^2 Q\right) + \frac{\partial}{\partial z}\left(C\chi^2 Q\right) - \frac{\partial}{\partial z}\left(\chi C F_\lambda\right)\end{gathered} \qquad \text{...(12)}$$

where $\xi = 2v/r + f$ is twice the local absolute angular velocity and $\zeta = (1/r)(\partial(rv)/\partial r)$ is the vertical component of relative vorticity at radius r. Further algebraic details of the derivation are given in Bui et al. (2009). The equation is a linear elliptic partial differential equation for ψ at any instant of time, when the radial and vertical structures of v and θ are known at that time, provided that the discriminant

$$D = -g\frac{\partial\chi}{\partial z}\left(\chi\xi(\zeta+f) + C\frac{\partial\chi}{\partial r}\right) - \left[\frac{\partial}{\partial z}(\chi C)\right]^2\ . \qquad \text{...(13)}$$

is positive. With a few lines of algebra one can show that $D = g\rho\xi\chi^3 P$, where

$$P = \frac{1}{\rho\chi^2}\left[\frac{\partial v}{\partial z}\frac{\partial\chi}{\partial r} - (\zeta+f)\frac{\partial\chi}{\partial z}\right] \qquad \text{...(14)}$$

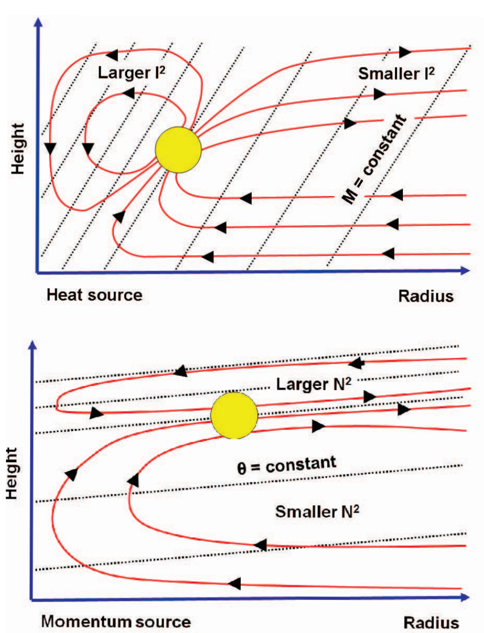
[5] Smith et al. (2005) show that the SE-equation emerges also from the time derivative of the toroidal vorticity equation when the time rate-of-change of the material derivative of potential toroidal vorticity, $\eta/(r\rho)$, is set to zero. Here $\eta = \partial u/\partial z - \partial w/\partial r$ is the toroidal, or azimuthal, component of relative vorticity.

is the Ertel potential vorticity (Shapiro and Montgomery, 1993).

Since the SE-equation (12) is a linear differential equation, the solution for the transverse streamfunction can be obtained by summing the solutions forced individually by the radial and vertical derivative of the diabatic heating rate and the vertical gradient of the azimuthal momentum forcing, respectively. Suitable boundary conditions on ψ are obtained using the relation- ship between ψ and the velocity components of the trans- verse circulation, viz., Eqn. (11). Solutions for a point source of diabatic heating in the tropical cyclone context were presented by Shapiro and Willoughby (1982).

Examples of the solution of (12) are shown in Fig. 1 which illustrates the secondary circulation induced by point sources of heat and absolute angular momentum in a balanced, tropical-cyclone-like vortex in a partially bounded domain (Willoughby 1995). Willoughby notes that, in the vicinity of the imposed heat source, the secondary circulation is congruent to surfaces of constant absolute angular momentum and is thus primarily vertical. In order to maintain a state of gradient and hydrostatic balance and slow evolution, the flow through the source is directed generally so as to oppose the forcing. Since the vortex is assumed to be stably stratified in the large, the induced flow through the heat source causes an adiabatic cooling tendency that tries to oppose the effects of heating. In the vicinity of the imposed momentum source (assumed positive in Fig. 1, corresponding to an eddy-induced cyclonic torque in the upper troposphere), the transverse circulation is congruent to surfaces of constant potential temperature and is primarily outwards. The resulting streamlines in either case form two counter-rotating cells of circulation (or gyres) that extend outside the source. There is a strong flow between these gyres and a weaker return flow on the outside. The flow emerges from the source, spreads outwards through a large volume surrounding it, and converges back into it from below. Thus, compensating subsidence surrounds heat-induced updrafts and compensating inflow lies above and below momentum-induced outflow.

Fig. 1. Secondary circulation induced in a balanced vortex by a heat source (upper panel) and a cyclonic momentum source (lower panel) in regions with different magnitudes of inertial stability, I^2 and thermodynamic stability N^2, and baroclinicity S^2. The strong motions through the source follow lines of constant angular momentum for a heat source and of constant potential temperature for a momentum source. Adapted from Willoughby (1995).

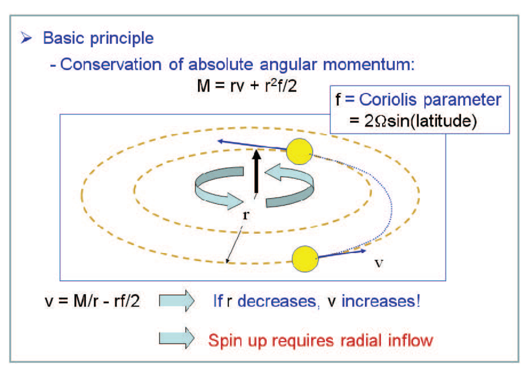


The radial scale of the gyres is controlled by the local Rossby length, NH/I, where $N^2 = (g/\theta)(\partial\theta/\partial z)$ or $-(g/\chi)(\partial\chi/\partial z)$, is a measure of the static stability for vertically displaced air parcels (N being the Brunt-Väisälä frequency), $I^2 = \xi\,(f + \zeta)$ is a measure of the inertial (centrifugal) stability for horizontally displaced rings of fluid assumed initially in hydrostatic and gradient wind balance, and H is the depth of the overturning layer. Thus, the ratio of the horizontal to vertical scales is proportional to N/I. From the foregoing discussion it follows that a heat source located in the middle troposphere induces inflow in the lower troposphere and outflow in the upper troposphere beyond the radius of the source (Fig. 1(a)). At radii inside that of the heat source, a reversed cell of circulation is induced with subsidence along and near the axis. The situation is similar for more realistic distributions of diabatic heating (Bui et al. 2009). Similarly, in order to maintain a state of balanced flow a momentum sink associated with surface friction distributed through a frictional boundary layer will induce inflow in the boundary layer and outflow above the layer.

The key element of vortex spin-up in an axisymmetric setting can be illustrated from the equation for absolute angular momentum per unit mass

$$M = rv + \frac{1}{2}fr^2 \qquad \text{...(15)}$$

Fig. 2. Schematic diagram illustrating the spin up associated with the inward-movement of M surfaces.



given by

$$\frac{\partial M}{\partial t} + u\frac{\partial M}{\partial r} + w\frac{\partial M}{\partial z} = F \qquad ...(16)$$

where $F = rF_{\lambda}$ represents the torque per unit mass acting on a fluid parcel in association with frictional or (unresolved) turbulent forces, or those associated with non-axisymmetric eddy processes.[6] This equation is, of course, equivalent to the axisymmetric tangential momentum equation (Eqn. (8)). If $F = 0$, then M is materially conserved, i.e. $DM/Dt = 0$, where $D/Dt = \frac{\partial}{\partial t} + u\frac{\partial}{\partial r} + w\frac{\partial}{\partial z}$ is the material derivative following fluid particles in the axisymmetric flow. Since M is related to the tangential velocity by the formula:

$$v = \frac{M}{r} - \frac{1}{2}fr, \qquad ...(17)$$

we see that, when M is materially-conserved, both terms in this expression lead to an increase in v as r decreases and to a decrease in v when r increases. Thus *a pre-requisite for spin-up in an inviscid axisymmetric flow is azimuthal-mean radial inflow*. Conversely, as air parcels move outwards, they spin more slowly. An alternative, but equivalent interpretation for the material rate-of-change of the mean tangential wind in an axisymmetric inviscid flow follows directly from Newton's second law (see Eqn. 8) in which the sole force is the generalized Coriolis force, $-u(v/r + f)$, where u is the mean radial velocity component, associated with the mean radial component of inflow. In regions where frictional forces are appreciable, F is negative definite (provided that the tangential flow is cyclonic relative to the earth's local angular rotation, $v/r > -f$), and M decreases following air parcels. We will show in 'Two spin-up mechanisms' later in this article that friction plays an important dynamical role in the spin-up of a tropical cyclone.

A balance theory for spin-up

As intimated earlier in this article, the assumption that the flow is balanced everywhere paves the way for a method to solve an initial-value problem for the slow evolution of an axisymmetric vortex forced by sources of heat (Q) and tangential momentum (F_{λ}). Given an initial tangential wind profile, $v_i(r, z)$, and some environmental density sounding $\rho_o(z)$, one would proceed using the following basic steps:

- (1) solve Eqn. (3) for the initial balanced density and pressure fields corresponding to v_i.
- (2) solve the SE-equation. (12) for ψ.
- (3) solve for the velocity components u and w of the overturning circulation using Eqn. (11).
- (4) predict the new tangential wind field using Eqn. (8) at a small time Δt.
- (5) repeat the sequence of steps from item (1).

The method is straightforward to implement. Examples are given by Sundqvist (1970), Schubert and Alworth (1987) and Möller and Smith (1994). For strong tropical cyclones, the boundary layer and upper-tropospheric outflow region generally develop regions of zero or negative discriminant ($D < 0$). A negative discriminant implies the development of regions supporting symmetric instability and technically speaking the global balance solution breaks down. Nevertheless, it is often possible to advance the balance solution forward in time to gain a basic understanding of the long-time balance flow structure. If, for example, the symmetric instability regions remain localised and do not extend throughout the mean vortex, one may apply a regularization procedure to keep the SE-equation elliptic and thus invertible (see 'An axisymmetric balance view of spin-up' later in this article).[7]

Boundary-layer dynamics and departures from gradient wind balance

We refer to the tropical cyclone boundary layer as the shallow region of strong inflow adjacent to the ocean surface, which is typically 500 m to one km deep and in which the effects of surface friction are important. A scale analysis of the equations of motion indicates that the radial pressure gradient force of the flow above the boundary layer is transmitted approximately unchanged through the boundary layer to the surface (see, e.g. Vogl and Smith 2009). However, beyond some radius outside the radius of maximum gradient wind,[8] the centrifugal and Coriolis forces near the ocean surface are reduced because of the frictional retardation of the tangential wind (Fig. 3). The resulting imbalance of the radial pressure gradient force and the centrifugal and Coriolis forces implies a radially inward *agradient force*, F_a , that generates an inflow near the surface. The agradient force is defined as the difference between the local radial pressure gradient and the sum of the centrifugal and Coriolis forces per unit mass, i.e. $F_a = -(1/\rho)(\partial p/\partial r) + (v^2/r + fv)$, where the various quantities are as defined earlier. If $F_a = 0$, the tangential flow is in exact gradient wind balance; if $F_a < 0$, this flow is *subgradient* and if $F_a > 0$ it is *supergradient*. As we have defined it, the agradient force is a measure of the effective radial pressure gradient force, but an alternative definition might include frictional forces also.

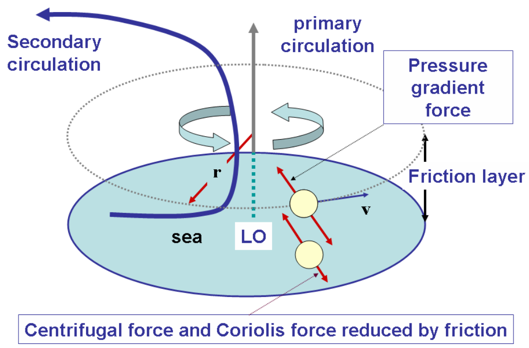
The foregoing considerations naturally motivate a

[6] Apart from a factor of 2π, M is equivalent to Kelvin's circulation Γ for a circle of radius r enclosing the centre of circulation, i.e. $2\pi M = \Gamma = \oint \mathbf{v_{abs}} \cdot \mathbf{dl}$, where $\mathbf{v}_{abs}$ is the absolute velocity and $\mathbf{dl}$ is a differential line segment along the circle. The material conservation of M is equivalent to Kelvin's circulation theorem.

[7] An alternative approach is to formulate the balanced evolution in terms of moist equivalent potential temperature instead of dry potential temperature. A particularly elegant method within this framework is to assume that air parcels rising out of the boundary layer materially conserve their equivalent potential temperature and that the analogous discriminant for the moist SE-equation is everywhere zero, with an implied zero moist potential vorticity. Such an approach, together with a crude slab boundary layer representation, is employed in a class of time-dependent models that underpin the WISHE intensification paradigm discussed in 'The role of evaporation-wind feedback' and subsequent generalizations reviewed later in this article.

[8] The situation in the inner region is more complex as discussed below.

Fig. 3. Schematic diagram illustrating the agradient force imbalance in the friction layer of a tropical cyclone and the secondary circulation that it generates.



dynamical definition of the boundary layer. Since this layer arises because of the frictional disruption of gradient wind balance near the surface, we might define the boundary layer as the surface-based layer in which the inward-directed agradient force exceeds a specified threshold value. This dynamical definition is uncontroversial in the outer regions of a hurricane, where there is subsidence into the boundary layer, but it is questionable in the inner core region where boundary-layer air separates from the surface and is lofted into the eyewall clouds (Smith and Montgomery, 2010).

Because of the pattern of convergence within the boundary layer and the associated vertical velocity at its top, the layer exerts a strong control on the flow above it. In the absence of diabatic heating associated with deep convection, the boundary-layer would induce radial outflow in a layer above it and the vortex would spin down as air parcels move to larger radii while conserving their absolute angular momentum.[9] If the air is stably stratified, the vertical extent of the outflow will be confined. It follows that a requirement for the spin-up of a tropical cyclone is that the radial inflow in the lower troposphere induced by the diabatic heating must more than offset the frictionally-induced outflow (e.g. Smith 2000).

Where the boundary layer produces upflow, it plays an additional role by determining the radial distribution of absolute angular momentum, water vapour and turbulent kinetic energy that enter into the vortex above. This characteristic is an important feature of the spin-up in the three axisymmetric paradigms for tropical cyclone intensification to be discussed. In particular, the source of moisture that fuels the convection in the eyewall enters the boundary layer from the ocean surface, whereupon the boundary layer exerts a significant control on the preferred areas for deep convection.

As the vortex strengthens, the boundary-layer inflow becomes stronger than the balanced inflow induced directly by the diabatic heating and the tangential frictional force as discussed earlier in this article. This breakdown of balance dynamics provides a pathway for air parcels to move inwards quickly and we can envisage a scenario in which the boundary layer takes on a new dimension. Clearly, if M decreases less rapidly than the radius following an inward moving air parcel, then it follows from Eqn. (17) that the tangential wind speed will increase following the air parcel. Alternatively, if spiralling air parcels converge quickly enough so that the generalised Coriolis force acting on them exceeds the tangential component of frictional force, the tangential winds can increase with decreasing radius. These considerations raise the possibility that the tangential wind in the boundary layer may ultimately exceed that above the boundary layer in the inner region of the storm. This possibility will be shown to be a reality later in this article.

The CISK-paradigm

In a highly influential paper, Charney and Eliassen (1964) proposed an axisymmetric balance theory for the cooperative interaction between a field of deep cumulus clouds and an incipient, large-scale, cyclonic vortex. A similar theory, but with a different closure for deep convection was proposed independently by Ooyama (1964). These theories highlighted the role of surface friction in supporting the amplification process. They were novel because friction was generally perceived to cause a spin down of an incipient vortex. In their introduction, Charney and Eliassen state: 'Friction performs a dual role; it acts to dissipate kinetic energy, but because of the frictional convergence in the moist surface boundary layer, it acts also to supply latent heat energy to the system.' This view has prevailed until very recently (see 'An axisymmetric view of the new paradigm' later in this article).

The idea of cooperative interaction stems from the closure assumption used by Charney and Eliassen that the rate of latent heat release by deep cumulus convection is proportional to the vertically-integrated convergence of moisture through the depth of the troposphere, which occurs mainly in the boundary layer. Recall from earlier in this article that, in a balanced vortex model where the contribution of friction to the secondary circulation is initially relatively small, the strength of the azimuthal mean overturning circulation is proportional to the radial gradient of the net diabatic heating rate. In a deep convective regime in which the diabatic heating rate, and therefore its radial gradient, are a maximum in the middle to upper troposphere, this balanced circulation is accompanied by inflow below the heating maximum and outflow above it. At levels where there is inflow, the generalised Coriolis force acting on the inflow amplifies the tangential wind. The increased tangential wind at the top of the boundary layer leads to an increase of the frictional inflow in the boundary

[9]This mechanism for vortex spin down involving the frictionally-induced secondary circulation is the primary one in a vortex at high Reynolds' number and greatly overshadows the direct effect of the frictional torque on the tangential component of flow in the boundary layer (Greenspan and Howard 1963).

and therefore to an increase of moisture convergence in the inflow layer (see Fig. 4). The closure that relates the latent heating to the moisture convergence then implies an increase in the heating rate and its radial gradient, thereby completing the cycle.

Charney and Eliassen constructed an axisymmetric, quasi-geostrophic, linear model to illustrate this convective-vortex interaction process and found unstable modes at sub-synoptic scales shorter than a few hundred kilometres. In order to distinguish this macro instability from the conventional conditional instability that leads to the initiation of individual cumulus clouds, it was later named Conditional Instability of the Second Kind (CISK) by Rosenthal and Koss (1968). A clear picture of the linear dynamics of the intensification process was provided by Fraedrich and McBride (1989), who noted that '... the CISK feedback is through the spin-up brought about by the divergent circulation above the boundary layer', as depicted here in the schematic in Fig. 4. Similar ideas had already been suggested much earlier by Ooyama (1969, left column, p18) in the context of a linear instability formulation with a different closure assumption.[10]

For many years subsequently, this so-called CISK-theory enjoyed wide appeal by tropical meteorologists. Indeed, the theory became firmly entrenched in the teaching of tropical meteorology and in many notable textbooks (e.g Holton, 1992, section 9.7.2; James, 1994, pp279–81). The first papers on the topic stimulated much subsequent research. A list of references is given by Fraedrich and McBride (1989).

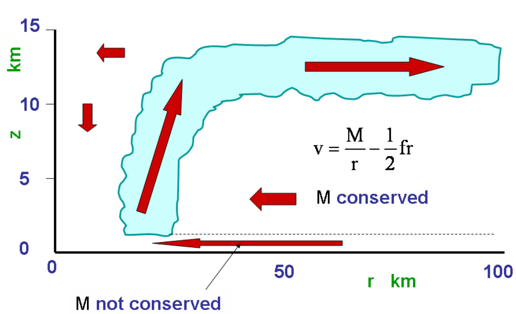
Despite its historical significance and influence, the CISK theory has attracted much criticism. An important contribution to the debate over CISK is the insightful paper by Ooyama (1982), which articulated the cooperative intensification paradigm for the spin-up process discussed in the next section. Ooyama noted that the CISK closure for moist convection is unrealistic in the early stage of development. The reason is that there is a substantial separation of horizontal scales between those of deep convective towers and the local Rossby length for the mean vortex (as defined earlier in this article). Therefore, during this stage, the convection is not under 'rotational control' by the parent vortex. Indeed, Ooyama cautioned that the CISK closure and variants thereof were little more than convection in disguise, exhibiting the largest growth rates at the smallest horizontal scales.

Later in the 1980s and 1990s, the CISK theory was critiqued in a number of papers by Emanuel (1986, hereafter E86), Raymond and Emanuel (1993), Emanuel et al. (1994), Craig and Gray (1996), Ooyama (1997) and Smith (1997). Raymond and Emanuel op. cit. gave an erudite discussion of the issues involved in representing cumulus clouds in numerical models. They recalled that the premise underlying all physical parameterizations is that some aspect of the chaotic microscale process is in statistical equilibrium with the macroscale system. They noted also that the statistical equilibrium assumption implies a particular chain of causality, to wit: 'viscous stresses *are caused by* changes in the strain rate; turbulence *is caused by* instability of the macroscale flow; and convection *is caused by* conditional instability. Conditional instability is quantified by the amount of Convective Available Potential Energy (CAPE) in the macroscale system and convection, in turn, consumes this CAPE.' Raymond and Emanuel argued that the CISK closure implies a statistical equilibrium of water substance wherein convection is assumed to consume water (and not directly CAPE) at the rate at which it is supplied by the macroscale system. Indeed, they argued that the closure fundamentally violates causality because convection is not caused by the macroscale water supply.

Emanuel et al. (1994) pointed out that the CISK closure calls for the large-scale circulation to replenish the boundary-layer moisture by advecting low-level moisture (and hence CAPE) from the environment, arguing that it completely overlooks the central role of surface moisture fluxes in accomplishing the remoistening. He went on to argue that, based on CISK theory, cyclone intensification would be just as likely to occur over land as over the sea, contrary to observations. This particular criticism seems a little harsh as Charney and Eliassen did say on p71 of their paper: 'We have implicitly assumed that the depression forms over the tropical oceans where there is always a source of near-saturated air in the surface boundary layer. However, we shall ignore any flux of sensible heat from the water surface'.

[10] Ooyama's reference to the 'lower layer' refers to the lower troposphere above the boundary layer, see his Fig. 1.

Fig. 4. Schematic of the CISK-paradigm and of the cooperative intensification paradigm of tropical cyclone intensification. The basic tenet is that, in an axisymmetric-mean sense, deep convection in the inner-core region induces inflow in the lower troposphere. Above the frictional boundary layer, the inflowing air conserves its absolute angular momentum and spins faster. Strong convergence of moist air mainly in the boundary layer provides 'fuel' to maintain the convection. In the CISK-paradigm, the rate of latent heat release by deep cumulus convection is proportional to the vertically-integrated convergence of moisture through the depth of the troposphere. The bulk of this moisture convergence occurs in the boundary layer. In the cooperative intensification paradigm the representation of latent heat release is more sophisticated.



Another concern is that the heating representation assumed in the CISK theory is not a true 'sub-grid scale' parameterisation of deep convection in the usual sense, but is simply a representation of moist pseudo-adiabatic ascent (Smith 1997). Furthermore, the assumption that a stronger overturning circulation leads to a greater diabatic heating, while correct, misses the point since the adiabatic cooling following the air parcels increases in step. In other words, the pseudo-equivalent potential temperature, θ_e, of ascending air is materially conserved and is determined by the value of θ_e where the ascending air exits the boundary layer (E86). Thus, the radial gradient of virtual potential temperature at any height in the cloudy air will not change unless there is a corresponding change in the radial gradient of θ_e in the boundary layer.

The cooperative-intensification paradigm

Although Charney and Eliassen's seminal paper continued to flourish for a quarter of a century, soon after publishing his 1964 paper, Ooyama recognized the limitations of the linear CISK paradigm. This insight led him to develop what he later termed a cooperative intensification theory for tropical cyclones (Ooyama 1982, section 4; Ooyama 1997, section 3.2), but the roots of the theory were already a feature of his simple nonlinear axisymmetric balance model for hurricane intensification (Ooyama 1969). The 1969 study was one of the first successful simulations and consistent diagnostic analyses of tropical cyclone intensification.[11]

The cooperative intensification theory assumes that the broad-scale aspects of a tropical cyclone may be represented by an axisymmetric, balanced vortex in a stably stratified, moist atmosphere. The basic mechanism was explained by Ooyama (1969, p18) as follows. 'If a weak cyclonic vortex is initially given, there will be organised convective activity in the region where the frictionally-induced inflow converges. The differential heating due to the organised convection introduces changes in the pressure field, which generate a slow transverse circulation in the free atmosphere in order to re-establish the balance between the pressure and motion fields. If the equivalent potential temperature of the boundary layer is sufficiently high for the moist convection to be unstable, the transverse circulation in the lower layer will bring in more absolute angular momentum than is lost to the sea by surface friction. Then the resulting increase of cyclonic circulation in the lower layer and the corresponding reduction of the central pressure will cause the boundary-layer inflow to increase; thus, more intense convective activity will follow.'

In Ooyama's model, the 'intensity of the convective activity' is characterised by a parameter η, where $\eta - 1$ is proportional to the difference between the moist static energy in the boundary layer and the saturation moist static energy in upper troposphere. Physically, $\eta - 1$ is a measure of the degree of local conditional instability for deep convection in the vortex. Ooyama noted that, in his model, as long as $\eta > 1$, the positive feedback process between the cyclonic circulation and the organized convection will continue. The feedback process appears to transcend the particular parameterisation of deep convection used by Ooyama. Although Ooyama took the cloud-base mass flux to be equal to the convergence of resolved-scale mass flux in the boundary layer, this restriction may be easily relaxed (Zhu et al. 2001).

Ooyama's model contained a simple bulk aerodynamic representation of the surface moisture flux (his Eqn. [7.4]) in which the flux increases with surface wind speed and with the degree of air-sea moisture dis-equilibrium. Although Ooyama recognised the need for such fluxes for intensification, he did not discuss the consequences of their wind speed dependence. However, he did point out that as the surface pressure decreased sharply with decreasing radius in the inner-core region, this would lead to a concomitant sharp increase in the saturation mixing ratio, q_s^*. The increase in q_s^*, which is approximately inversely proportional to the pressure, may boost the boundary layer moisture provided that the mixing ratio of near-surface air doesn't increase as fast (see the discussion below Eqn. (7.4) in Ooyama [1969]).

Willoughby (1979) noted that without condensational heating associated with the convection, air converged in the boundary layer would flow outwards just above the boundary layer instead of rising within the clouds and flowing out near the tropopause. In other words, for intensification to occur, the convectively-induced convergence must be large enough to more than offset the frictionally-induced divergent outflow above the boundary layer (Raymond et al. 1998, Marin et al. 2009, Smith 2000).

The foregoing feedback process differs from Charney and Eliassen's CISK paradigm in that the latter does not explicitly represent the oceanic moisture source and it assumes that the latent heat release within the region of deep convection is proportional to the vertically-integrated radial moisture flux. Unlike the CISK theory, the cooperative intensification theory does not represent a runaway instability, because, as the troposphere warms on account of the latent heat release by convection, the atmosphere becomes progressively less unstable to buoyant deep convection. Moreover, as the boundary-layer moisture increases, the degree of moisture disequilibrium at the sea surface decreases. If the near-surface air were to saturate, the surface moisture flux would be zero, irrespective of the wind speed.

Two aspects of both the CISK theory and Ooyama's cooperative intensification theory that have gained wide acceptance are that:

- the main spin-up of the vortex occurs via the convergence of absolute angular momentum above the boundary layer; and
- the boundary layer plays an important role in converging

[11]In our view, Craig and Gray's (1996) categorisation of the Ooyama (1969) model as being a version of CISK was convincingly refuted by Ooyama (1997).

moisture to sustain deep convection, but its dynamical role is to oppose spin-up.

In addition, Ooyama's theory recognises explicitly the need for a flux of moisture from the ocean to maintain a degree of convective instability, which is needed for the intensification process.

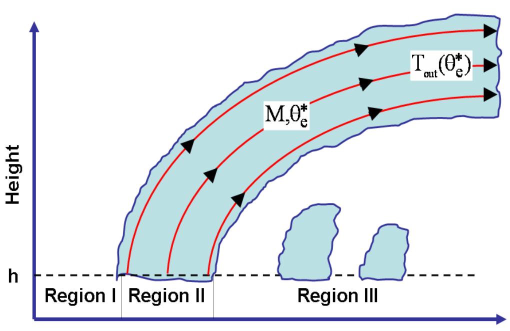
An idea within the framework of the cooperative intensification theory is the 'convective ring' model of intensification summarised by Willoughby (1995, his sections 2.2.2 and 2.5.2; see also Willoughby 1990b). This model invokes the axisymmetric balance theory discussed earlier in this article and argues based on composites of airborne radar reflectivity that intensification proceeds by the inward propagation of ring-like convective structures.

The WISHE intensification paradigm

A steady-state hurricane model

In what has become a highly influential paper, E86 developed a steady-state axisymmetric balance model for a mature tropical cyclone that subsequently led to a major paradigm shift in the theory for the intensification of these storms. In order to discuss this new paradigm, it is necessary to review some salient features of the E86 model. Briefly, the hurricane vortex is assumed to be steady and circularly symmetric about its axis of rotation. The boundary layer is taken to have uniform depth, h, and is divided into three regions as shown in Fig. 5. Regions I and II encompass the eye and eyewall, respectively, while Region III refers to that beyond the radius, r_m, of maximum tangential wind speed, v_m, at the top of the boundary layer.

Fig. 5. Schematic diagram of Emanuel's 1986 model for a mature steady-state hurricane. The boundary layer is assumed to have constant depth h and is divided into three regions as shown: the eye (Region I), the eyewall (Region II) and outside the eyewall (Region III) where spiral rainbands and shallow convection emanate into the vortex above. The absolute angular momentum per unit mass, M, and equivalent potential temperature, θ_e, of an air parcel are conserved after the parcel leaves the boundary layer and ascends in the eyewall cloud. In the steady model, these parcel trajectories are stream surfaces of the secondary circulation along which M and θ_e are materially conserved. The precise values of these quantities depend on the radius at which the parcel exits the boundary layer. The model assumes that the radius of maximum tangential wind speed, r_m, is located at the outer edge of the eyewall cloud. As noted in footnote 12, recent observations indicate it is closer to the inner edge.



E86 takes the outer radius of Region II to be r_m on the basis that precipitation-driven downdrafts may be important outside this radius.[12] The tangential wind field is assumed to be in gradient and hydrostatic balance everywhere, including the boundary layer.[13] Moreover, upon exiting the boundary layer, air parcels are assumed to rise to the upper troposphere conserving their M and saturation moist entropy, s^* (calculated pseudo-adiabatically[14]). Above the boundary layer, the surfaces of moist entropy and absolute angular momentum are assumed to flare outwards with height and become almost horizontal in the upper troposphere. For an appraisal of some aspects of the steady-state model, see Smith et al. (2008) and Bryan and Rotunno (2009).

Time-dependent extensions of the E86 model were developed for investigating the intensification process (Emanuel 1989, 1995 and 1997: hereafter E97). These models were formulated in the context of axisymmetric balance dynamics and employ potential radius coordinates. The potential radius, R, is defined in terms of the absolute angular momentum by $\sqrt{2M/f}$ and is the radius to which an air parcel must be displaced, conserving M, so that its tangential wind speed vanishes.

The role of evaporation-wind feedback

The new paradigm for intensification that emerges from the time-dependent models invokes a positive feedback between the near-surface wind speed and the rate of evaporation of water from the underlying ocean, *which depends on the wind speed* (Emanuel et al. 1994, section 5a; Emanuel 2003). Emanuel's time-dependent models have broadly the same ingredients as Ooyama's 1969 model (e.g. gradient wind and hydrostatic balance, wind-speed-dependent surface moisture fluxes), but the closure for moist convection is notably different by assuming undilute pseudo-adiabatic ascent along surfaces of absolute angular momentum rather than upright ascent with entrainment of

[12]Contrary to Emanuel's assumption in this figure, observations show that r_m is located well inside the outer edge of the eyewall (e.g. Marks et al. 2008, their Fig. 3).

[13]Although E86 explicitly assumes gradient wind balance at the top and above the boundary layer (p586: 'The model is based on the assumptions that the flow above a well-mixed surface boundary layer is inviscid and thermodynamically reversible, that hydrostatic and gradient wind balance apply ...'), in the slab formulation therein the M and tangential velocity of the slab of air exiting the boundary layer is assumed to equal that of the flow at the top of the boundary layer. Since the swirling wind field at the top of the boundary layer is assumed to be in gradient wind balance (op. cit.), it is a mathematical consequence that in the region where the air is rising out of the boundary layer the tangential velocity of the slab of air exiting the boundary layer must be in gradient wind balance also.

[14]Although the original formulation and subsequent variants thereof purported to use reversible thermodynamics, it has been recently discovered that the E86 formulation and its variants that make a priori predictions for the maximum tangential wind (Emanuel 1995, 1997, and Bister and Emanuel 1998) tacitly implied pseudo-adiabatic thermodynamics (Bryan and Rotunno, 2009)

middle tropospheric air. Later in this article, we examine carefully the semi-analytical time-dependent E97 model, which has been advanced as the essential explanation of tropical cyclone intensification (Emanuel 2003). To begin, we discuss the traditional view of the evaporation-wind feedback intensification mechanism known generically as WISHE (see footnote 1).

Although the evaporation of water from the underlying ocean has been long recognised as the ultimate energy source for tropical cyclones (Kleinschmidt 1951; Riehl 1954; Malkus and Riehl 1960; Ooyama 1969),[15] Emanuel's contributions with colleagues re-focused attention on the air-sea interaction aspects of the intensification process. Rather than viewing latent heat release in deep convective towers as the 'driving mechanism' for vortex amplification, this body of work showed that certain aspects of these storms could be understood in terms of a simple time-dependent model in which the latent heat release was implicit.

The new dimensions introduced by the WISHE paradigm were the positive feedback process between the wind-speed dependent moisture fluxes and the tangential wind speed of the broad-scale vortex, and the finite-amplitude nature of the instability, whereby the incipient vortex must exceed a certain threshold intensity for amplification to proceed. Emanuel and collaborators emphasised also the non-necessity of CAPE in the storm environment for intensification.

The evaporation-wind feedback intensification mechanism has achieved widespread acceptance in meteorology textbooks and other didactic material (e.g. Rauber et al. 2008, Holton 2004, Asnani 2005, Ahrens 2008, Holton and Hakim 2012, COMET course in Tropical Meteorology[16]), tropical weather briefings and the current literature (Lighthill 1998, Smith 2003, Molinari et al. 2004, Nong and Emanuel 2004, Montgomery et al. 2006, Terwey and Montgomery 2008, Braun et al. 2010). Indeed, the last four authors and others have talked about 'igniting the WISHE mechanism' after the vortex (or secondary maximum in the tangential wind) has reached some threshold intensity.

In his review paper Emanuel (2003) specifically describes the intensification process as follows: 'Intensification proceeds through a feedback mechanism wherein increasing surface wind speeds produce increasing surface enthalpy flux..., while the increased heat transfer leads to increasing storm winds.' In order to understand his ensuing description of the intensification process, one must return to the E97 paper because two of the key parameters, α and β, in Eqn. (10) of the 2003 paper are undefined. 'A simple model of the intensification process' later in this article provides a careful examination of the corresponding equation (Eqn. [20]) in E97 and the underlying assumptions employed therein.

[15]Technically speaking, the sun is the ultimate energy source for all atmospheric and oceanic motions.
[16]http://www.comet.ucar.edu

We have a number of issues with some of the popular articulations of the putative WISHE spin-up mechanism (e.g. that of Holton and Hakim op. cit.) and present here our own interpretation of the mechanism, summarizing that given in Montgomery et al. (2009).

The WISHE intensification mechanism in detail

The WISHE process for intensifying the inner-core tangential wind is illustrated schematically in Fig. 6. It is based on the idea that, except near the centre, the near-surface wind speed in a tropical cyclone increases with decreasing radius. This increase is typically accompanied by an increase in near-surface specific humidity, which leads to a negative radial gradient of θ_e there, and hence throughout the boundary layer by vertical mixing processes. On account of frictional convergence, air parcels in the inner core exit the boundary layer and are assumed to rise upwards and ultimately outwards into the upper troposphere. As they do so, they materially conserve their M and θ_e values, carrying with them an imprint of the near-surface radial gradients of M and θ_e into the interior of the vortex. Since the rising air rapidly saturates, the radial gradient of θ_e implies a negative radial gradient of virtual temperature. Thus, in the cloudy region, at least, the vortex is warm cored. As air parcels move outwards conserving M they spin more slowly about the rotation axis of the storm, which, together with the positive radial gradient of M, explains the observed decrease of the tangential wind speed with height. In the eyewall region it is assumed that all the streamlines emanating from the boundary layer asymptote to the undisturbed environmental θ_e surfaces, thereby matching the saturation θ_e of the streamlines. It is assumed also that, at large radii, these surfaces exit the storm in the lower stratosphere, where the absolute temperature is approximately constant.

Now suppose that for some reason the negative radial gradient of specific humidity increases in the boundary layer. This would increase the negative radial gradient of θ_e, both in the boundary layer and in the cloudy region of the vortex aloft, and thereby warm the vortex core in this region. Assuming that the vortex remains in thermal wind balance during this process, the negative vertical shear of the tangential wind will increase. E86 presents arguments to show that this increase leads to an increase in the maximum tangential wind speed *at the top of the boundary layer* (see, e.g. Montgomery et al. 2006, Eqn. A1). Assuming that this increase translates to an increase in the surface wind speed, the *surface* moisture flux will increase, thereby increasing further the specific humidity. However, this increase in specific humidity will reduce the thermodynamic disequilibrium in specific humidity between the near-surface air and the ocean surface, unless the saturation specific humidity at the sea surface temperature increases in step. The increase in wind speed could help maintain this disequilibrium if there is an associated reduction in surface pressure. (Recall from earlier in this article that the saturation specific humidity at the sea surface temperature is approximately inversely proportional

to the air pressure). If the specific humidity (and θ_e) in the boundary layer increases further, it will lead to a further increase in tangential wind speed and so on.[17]

WISHE and the cooperative intensification theory

Our discussion of the spin-up mechanism in the previous paradigms highlighted the role of convergence of absolute angular momentum in the lower troposphere. The arguments articulated earlier in the previous subsection are silent about the role of the secondary circulation in the vortex-scale amplification process and, because they assume thermal wind balance, the buoyancy of the warm core, or more precisely the system buoyancy, cannot be invoked to 'drive' the secondary circulation. Indeed, cloud updrafts in this idealization have no explicit local buoyancy.[18] In this light, then, it is natural to ask how the spin-up actually occurs in the WISHE theory? To answer this question we examine fur- ther the time-dependent E97 model referred to in 'The role of evaporation-wind feedback' earlier in this article. Recall that the E97 model is founded on the idea that spin-up is controlled by the thermodynamics of the boundary layer and makes the key assumptions that:

1. The vortex is in gradient wind balance[19]; and
2. Above the boundary layer, the moist isentropes and M surfaces are congruent and flare out to large radius.

In the steady-state model of E86, the flaring out of the M- and θ_e-surfaces implies that there is outflow everywhere above the boundary layer, since both these quantities are materially conserved. The situation in E97 model is less transparent because the model is formulated in potential radius coordinates (see 'The WISHE intensification paradigm') and even though these surfaces flare outwards in physical space, they will move radially in physical space as the vortex evolves. If the vortex intensifies at low levels above the boundary layer, they must move inwards there (see 'Spin-up' earlier in this article), consistent with net inflow at these levels, and if the vortex decreases in intensity, they must move outwards.

The effects of latent heat release in clouds are implicit also in the E97 model, but the negative radial gradient of θ_e in the boundary layer is roughly equivalent to a negative radial gradient of diabatic heating in the interior, which, according to the balance concepts discussed earlier in 'The overturning circulation' will lead to an overturning circulation with inflow in the lower troposphere. Thus the spin-up above the boundary layer is entirely consistent with that in Ooyama's cooperative intensification theory. Because the tangential wind speed in the slab boundary is assumed to be equal to that at the top of the boundary layer (as in Ooyama's 1969 model), the spin-up in the boundary layer is consistent also with Ooyama's 1969 model. Unfortunately, the mathematical elegance of the formulation in E97 is achieved at the cost of making the physical processes of spin-up less transparent (at least to us!).

For the foregoing reasons we would argue that the differences between the E97 theory and Ooyama's cooperative intensification theory are perhaps fewer than is widely appreciated. In essence:

- The convective parameterisations are different. E97 assumes pseudoadiabatic ascent along outward-sloping M surfaces, while Ooyama (1969) uses an upright entraining plume model for deep convection.
- Emanuel gives explicit recognition to coupling between the surface enthalpy fluxes and the surface wind speed (although this coupling was included in Ooyama's 1969 model).
- E97 recognised the importance of convective downdrafts on the boundary layer thermodynamics and included a crude representation of these.

Emanuel would argue that a further difference distinguishing the WISHE paradigm from the others is that it does not require the existence of ambient CAPE, which is crucial in the CISK theory and is present in Ooyama's (1969) model. However, Ooyama (1997, section 3.3) noted that 'the initial CAPE had short memory...' and recalled that in his 1969 study he 'demonstrated that a cyclone vortex could not develop by the initial CAPE alone…' Further, Dengler and Reeder (1998) have shown that ambient CAPE is not necessary in Ooyama's model or extensions thereof.

One additional feature of the E97 model is its recognition of the tendency of the eyewall to develop a front in M and θ_e. This frontogenesis arises from the convergence of M and θ_e brought about by surface friction. A second feature of the model is a proposed relationship between the lateral mixing of M at the eyewall front and the rate of intensification.

---

[17] The mechanism as articulated here is quite different from that described by Kepert (2011, p13), who interprets WISHE in the context of a steady state vortex as 'The role of the surface enthalpy fluxes in making the expansion of the inflowing boundary layer air isothermal rather than adiabatic…' Kepert makes no mention of the necessity of the wind-speed dependence of the fluxes of latent and sensible heat and makes no distinction between dry and moist enthalpy in his discussion.

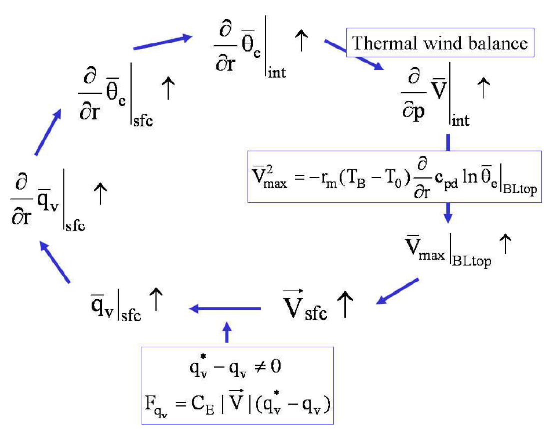
[18] A related property of the WISHE model is that any latent instability beyond that needed to overcome internal dissipation within cumulus clouds is believed unnecessary and thus irrelevant to the essential dynamics of tropical cyclone intensification (Rotunno and Emanuel 1987).

[19] While the statement on page 1019 of E97: 'We have assumed gradient and hydrostatic wind balance everywhere above the boundary layer...' might suggest otherwise, the use of a balanced boundary-layer formulation (Smith and Montgomery 2008) implies the *effective* assumption of gradient wind balance in the boundary layer also (Smith et al. 2008). Further, the assumption of gradient wind balance *everywhere* is a property of all three time-dependent models described in Emanuel (1989), Emanuel (1995), and E97, and is necessary to invoke an invertibility principle in which '... t*he entire rotational flow* [our emphasis] may be deduced from the radial distribution of θ_e in the boundary layer and the distribution of vorticity at the tropopause' (see first two complete paragraphs on page 1018 of E97). Note that, since the mathematical model represented by Eqn. (20) of E97 describes the dynamics of a slab boundary layer, the same scientific reasoning given in footnote 14 applies. Specifically, because the air at the top of the slab is assumed to be in gradient wind balance with tangential velocity V and because no discontinuities in the tangential velocity between the air rising out of the slab and that at its top are allowed, it is a logical consequence that the slab of air in the boundary layer must be in gradient wind balance.

Brakes on WISHE

As pointed out by Emanuel et al. (1994), the foregoing feedback mechanism is not a runaway process for the present climate and would cease if the boundary layer were to saturate. Moreover, the import of low entropy air into the boundary layer, for example by shallow convection, precipitation-induced downdrafts from deep convection, or vortex-scale subsidence, can significantly offset the moistening of the boundary layer by surface fluxes (e.g. Ooyama 1997). Indeed, Emanuel et al. (1994) argue that tropical cyclone amplification requires the middle troposphere to become nearly saturated on the mesoscale so that the downdrafts are weak enough not to negate the feedback process. However, we caution that recent evidence from state-of-the-art numerical cloud models suggests that the presence of mid-level dry air is not to strengthen convective downdrafts, but rather to reduce the strength of convective updrafts (James and Markowski 2010, Kilroy and Smith 2013).

Fig. 6. Schematic of the evaporation-wind intensification mechanism known as WISHE as articulated here. The pertinent variables are defined in the text and are otherwise standard. Thermal wind balance refers to the axisymmetric thermal wind equation in pressure coordinates that relates the radial gradient of azimuthal-mean θ_e to the vertical shear of the mean tangential velocity. The discussion assumes that, for some reason, the near-surface azimuthal mean mixing ratio is increased in the core region. This increase leads to an increase in the corresponding mean radial gradient and thereby to an increase in the mean radial gradient of θ_e throughout the boundary layer. The secondary circulation is imagined to loft this increase in θ_e into the vortex interior thereby warming the core region of the vortex. By the thermal wind relation and its corollary, obtained upon applying a Maxwell thermodynamic relation (presuming reversible thermodynamics, see footnote 14) and then integrating this equation along a surface of constant absolute angular momentum, this increased warmth is invoked to imply an increase in the mean tangential wind at the top of the vortex boundary layer. It is understood here that θ_e values at the boundary layer top correspond to saturated values (Emanuel 1986). Turbulent mixing processes in the boundary layer are assumed to communicate this wind speed increase to the surface, and thereby increase the potential for increasing the sea-to-air water vapour flux Fq_v. The saturation mixing ratio must increase approximately in step with the putative increase in near-surface q_v so as to maintain a thermodynamic disequilibrium. If this occurs then the surface mixing ratio will increase, thereby leading to a further increase in the mean tangential wind and so on.



Global energetics considerations point also to a brake on the intensification process and a bound on the maximum possible intensity. Recall that under normal circumstances the energy dissipation associated with surface friction scales as the cube of the tangential winds, while the energy input via moist entropy fluxes scales with the first power of the wind. Even if the boundary layer remains subsaturated, frictional dissipation will exceed the input of latent heat energy to the vortex from the underlying ocean at some point during the cyclone's intensification. This idea forms the basis for the so-called 'potential intensity theory' of tropical cyclones (E86, Emanuel 1988, 1995; E97; Bister and Emanuel 1998).[20]

A simple model of the intensification process

It should be noted that the schematic of the WISHE mechanism in Fig. 6 explains physically the sign of the tangential wind tendency in the core region, but it does not provide a means for quantifying the tendency. An elegant attempt to do this was developed by E97 in the form of an idealized axisymmetric, time-dependent model that would appear to include the WISHE intensification mechanism. A similar formulation has been used by Gray and Craig (1998) and Frisius (2006). E97 derived an expression for the time rate-of-change of tangential wind at the radius of maximum tangential wind (his Eqn. [20]) that equates the tangential wind tendency to the sum of three terms, two of which are always negative definite. The third term is positive only if the radial gradient of an 'ad hoc' function, *$\beta(r)$*, is sufficiently negative to offset the other two terms. The function β is introduced to 'crudely represent the effects of convective and large-scale downdrafts, which import low θ_e air into the subcloud layer' (E97, p1019, below Eqn. [16]). One weakness of the theory is the lack of a rigorous basis for the formulation of β. A second weakness is the effective

[20] At the present time it is thought there is an exception to this general argument when either the sea surface temperature is sufficiently warm or the upper tropospheric temperature is sufficiently cold, or some combination of the two prevails (Emanuel 1988, Emanuel et al. 1995). Under such conditions, the vortex is believed to be capable of generating enough latent heat energy via surface moisture fluxes to more than offset the dissipation of energy and a 'runaway' hurricane regime—the so-called 'hypercane regime'—is predicted. However, these predictions have been formulated only in the context of axisymmetric theory and it remains an open question whether hypercanes are indeed dynamically realizable in a three-dimensional context. Fortunately, the hypercane regime is not predicted for the present climate or predicted tropical climates of the future barring an unforeseen global extinction event (!). We will henceforth confine our attention to the intensification dynamics under current climate conditions.

assumption of gradient wind balance in the boundary layer[21] as discussed in footnote 19. Neither of these assumptions can be defended or derived from first principles.

In addition to the foregoing issues, we note a further limitation of this model. On page 1019 of E97, Emanuel draws attention to the '…crucial presence of downdrafts by reducing the entropy tendency there (outside the radius of maximum tangential wind, our insertion) by a factor β." However, the WISHE mechanism articulated in earlier in this article did not require downdrafts and moreover we show later in 'The role of rotating deep convection' that vortex intensification proceeds optimally in the pseudo-adiabatic case in which downdrafts are absent altogether!

In view of the foregoing limitations, it is difficult to regard this model as the archetype for tropical cyclone intensification.

A revised theory

Emanuel and Rotunno (2011) and Emanuel (2012) pointed out that the assumption referred to earlier in this article that the air parcels rising in the eyewall exit in the lower stratosphere in a region of approximate constant absolute temperature is poor. In the first of these papers, Emanuel and Rotunno op cit. found using an axisymmetric numerical model with (admittedly) unrealistically large values of vertical diffusion that 'the outflow temperature increases rapidly with angular momentum', i.e. the outflow layer is stably stratified. They hypothesized the important role of small-scale turbulence in determining this stratification and they postulated the existence of a critical gradient Richardson number. A diagnosis of the quasi-steady vortex offered some support for this hypothesis. Based on these results, Emanuel op. cit. presented a revised theory of the E97 model that avoids the assumption of a constant outflow temperature (and also the need to introduce the ad hoc β function noted previously). However, in three-dimensional model simulations with parameter settings that are consistent with recent observations of turbulence in hurricanes, Persing et al. (2013) show little support for the foregoing hypothesis. In these simulations, values of the gradient Richardson number are generally far from criticality with correspondingly little turbulent mixing in the upper level outflow region within approximately 100 km from the storm centre: only marginal criticality is suggested during the mature stage. This finding would suggest that the new theory of Emanuel (2012) is not a satisfactory theory for explaining tropical cyclone intensification in three dimensions.

In view of the limitations of the CISK and WISHE paradigms for tropical cyclone intensification presented above, we present now an overarching framework for understanding tropical cyclone intensification in high-resolution, three-dimensional, numerical model simulations and in reality.

[21]The scale analysis of the steady-state boundary layer equations performed by Smith and Montgomery (2008) is readily generalized to the intensification problem and shows that the gradient wind balance approximation cannot be justified for an intensifying tropical cyclone.

A new rotating-convective updraft paradigm

Satellite and radar observations have long suggested that tropical cyclones are highly asymmetric during their intensification phase. Only the most intense storms exhibit a strong degree of axial symmetry in their mature stages and even then, only in their inner-core region. Observations show also that rapidly developing storms are accompanied frequently by 'bursts' of intense convection (e.g. Gentry et al. 1970; Black et al. 1986; Marks et al. 1992; Molinari et al. 1999), which one would surmise possess significant local buoyancy and are accompanied by marked flow asymmetries. These are reasons alone to query the applicability of purely axisymmetric theories to the intensification process and a series of questions immediately arise:

1. How does vortex intensification proceed in three-dimensional models and in reality?
2. Are there fundamental differences between the intensification process in a three-dimensional model and that in an axisymmetric model?
3. Can the evolution of the azimuthally-averaged fields in three-dimensional models be understood in terms of the axisymmetric paradigms for intensification described in previous sections?
4. If not, can the axisymmetric paradigms be modified to provide such understanding?

These questions are addressed in the remainder of this review.

There have been many three-dimensional numerical-model studies of vortex amplification in the prototype problem for tropical cyclone intensification on an f-plane, described in the Introduction. These studies can be divided into five groups:

- those using hydrostatic models with cumulus parameterisation (e.g. Kurihara and Tuleya 1974);
- those using minimal hydrostatic models, with or without cumulus parameterisation (e.g. Zhu et al. 2001; Zhu and Smith 2002, 2003, Shin and Smith 2008);
- those using hydrostatic models with explicit micro-physics (e.g. Wang 2001, 2002a, 2002b);
- those using nonhydrostatic models with simplified physics (Nguyen et al. 2008, Montgomery et al. 2009, Persing et al. 2013); and
- those using nonhydrostatic models with sophisticated representations of physical processes (e.g. Wang 2008, Terwey and Montgomery 2009, Hill and Lackman 2009).

There have been investigations also of the analogous intensification problem on a β-plane, which is a prototype problem for tropical cyclone motion (e.g. Flatau et al. 1994; Dengler and Reeder 1997; Wang and Holland 1996a, 1996b).[22] In this scenario, initially symmetric vortices develop asymmetries from the very start of the integration on account of the 'β-effect', which is most easily understood

[22]There have been many more studies of this problem in a barotropic context, but our interest here is focussed on baroclinic models with at least three vertical levels to represent the effects of deep convection.

in terms of barotropic dynamics. In a barotropic vortex, air parcels conserve their absolute vorticity and, as they move polewards on the eastern side of the vortex, their relative vorticity becomes more anticyclonic. Conversely, air parcels that move equatorwards on the western side become more cyclonic. As time proceeds, this process leads to a vorticity dipole asymmetry that has fine-scale radial structure in the inner-core, where the radial gradient of angular velocity is large, but has a broad coherent structure at large radii where this gradient is small (Chan and Williams 1989, Smith et al. 1990, Smith and Ulrich 1993 and references). The asymmetric flow associated with this dipole leads to a westward and poleward motion across the vortex core and is influenced mostly by the coherent vorticity asymmetry at large radii (e.g. Smith et al. 1990. p351). In a baroclinic vortex, it is the potential vorticity that is conserved, except where there is diabatic heating associated with convection, but the mechanism of formation of the large-scale vorticity asymmetry is essentially as described above.

While the formation of flow asymmetries is to be expected on a β-plane, it may seem surprising at first sight that appreciable inner-core flow asymmetries emerge also in three-dimensional simulations that start with an axisymmetric vortex on an f -plane. It is the latter aspect that is the main focus of this section.

The development of flow asymmetries on an f-plane was already a feature of the early calculations by Kurihara and Tuleya (1974), but these authors did not consider these asymmetries to be remarkable, even though their problem as posed was, in essence, axisymmetric. Flow asymmetries were found also in simulations using a minimal (three-layer) model for a tropical cyclone that was developed in a series of papers by Zhu et al. (2001) and Zhu and Smith (2002, 2003). That model was designed initially to examine the sensitivity of tropical cyclone intensification to different convective parameterization schemes in the same model and in a configuration that was simple enough to be able to interpret the results. It came as a surprise to those authors that the vortex in their calculations developed flow asymmetries and the cause of these was attributed primarily to the coarse vertical resolution and the use of a Lorenz grid for finite differencing in the three layers. In the last paper (Zhu and Smith 2003), it was shown that the amplitude of asymmetries could be reduced by using a Charney-Phillips grid pattern in the vertical.

The role of rotating deep convection

Subsequent attempts to understand the development and evolution of the flow asymmetries in the prototype intensification problem on an f-plane were described by Nguyen et al. (2008) using the non-hydrostatic model, MM5,[23] and Shin and Smith (2008) using the minimal model of Zhu et al. (2001). We describe below the essential features of vortex evolution found in the Nguyen et al study. The model was stripped down to what were considered to be the simplest representations of physical processes necessary, including the simplest explicit representation of moist processes that mimics pseudo-adiabatic moist thermodynamics and a simple bulk formulation of the boundary layer including air-sea exchange processes. The horizontal grid spacing in the main experiments was 5 km and there were 24 levels in the vertical (7 below 850 mb) giving a much higher resolution than Zhu et al. 's minimal model, which had a 20-km horizontal grid and only three vertical layers. The calculations were initialized with an axisymmetric, warm-cored, cloud-free vortex with a maximum tangential velocity of 15 m s^{-1}.

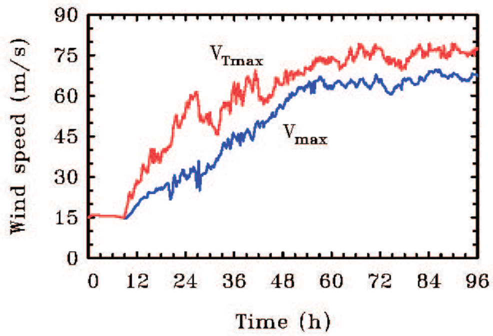
Figure 7 shows time-series of the maximum total horizontal wind speed, V_{Tmax}, and maximum azimuthally-averaged tangential wind component, V_{max}, at 900 hPa (approximately 1 km high). As in many previous experiments, there is a gestation period during which the vortex slowly decays because of surface friction, but moistens in the boundary layer because of evaporation from the underlying sea surface. During this period, lasting about 9 hours, the vortex remains close to axisymmetric. The imposition of friction from the initial instant leads to inflow in the boundary layer and outflow immediately above it, for the reasons discussed earlier in this artcicle. It is the outflow together with the conservation of absolute angular momentum that accounts for the initial decrease in the tangential wind speed (see 'Spin-up' earlier in this article).

The subsequent evolution is exemplified by Fig. 8 and Fig. 9, which show the patterns of vertical velocity and the vertical component of relative vorticity at the 850 mb level at selected times. The two left panels show the situation when rapid intensification begins. The inflowing air is moist and as it rises out of the boundary layer and cools, condensation occurs in some grid columns inside the radius of maximum tangential wind speed. Latent heat release in these columns initiates deep convective updrafts with vertical velocities up to 5 m s^{-1} at 850 mb. The updrafts form in an annular region inside the radius of the maximum initial tangential wind speed (135 km) and their distribution shows a dominant azimuthal wavenumber-12 pattern around the circulation centre.[24] The updrafts rotate cyclonically around the vortex centre and have lifetimes on the order of an hour. As they develop, they tilt and stretch the local vorticity field and an approximate ring-like structure of intense, small-scale, vorticity dipoles emerges (lower left panel of Fig. 8). The positive and negative components of the dipole are unequal with strong cyclonic vorticity and much weaker anticyclonic vorticity. Early in the intensification phase, the strongest updrafts lie approximately in between the vorticity dipoles whereas later, they are often approximately

[23]MM5 refers to the Pennsylvania State University-National Center for Atmospheric Research fifth-generation Mesoscale Model.

[24]It turns out that the number of updrafts that form initially increases as the horizontal resolution increases, but the subsequent evolutionary picture is largely similar.

Fig. 7. Time-series of maximum azimuthal-mean tangential wind component (V_{max}) and maximum total wind speed (V_{Tmax}) at 900 mb characterising vortex development in f -plane control experiment in M1.



collocated with the strong cyclonic anomalies. Following Nguyen et al. (2008), we refer to the cyclonically-rotating updrafts as 'vortical hot towers' (VHTs), a term first coined by Hendricks et al. (2004). However, we should caution that these updrafts need not penetrate all the way to the tropopause to greatly enhance the local ambient rotation (Wissmeier and Smith, 2011).

The development of the updrafts heralds a period lasting about two days during which the vortex intensifies rapidly, with V_{max} increasing at an average intensification rate of about 1 m s^{-1} h^{-1}. During this period, there are large fluctuations in V_{Tmax}, up to 15 m s^{-1}. Eventually, the vortex intensity reaches a quasi-steady state, in which V_{max} increases only slightly.

During the rapid intensification phase, the number of VHTs decreases from twelve at 9 h to no more than three by the end of the period of rapid intensification, but their structure remains spatially irregular and during some periods, the upward motion occupies a contiguous region around the vortex centre. The cyclonic vorticity anomalies associated with the VHTs grow horizontally in scale due to merger and axisymmetrization with neighbouring cyclonic vorticity anomalies. This upscale growth occurs in tandem with the convergence of like-sign vorticity into the circulation of the VHTs. They move slowly inwards also and become segregated from the anticyclonic vorticity anomalies, which move slowly outwards relative to them. As the anticyclonic anomalies move outwards they decrease in amplitude and undergo axisymmetrization by the parent vortex. Elements of the segregation process above the boundary layer are discussed by Nguyen et al. in their section 3.1.5.

The right panels of Fig. 8 show the situation at 24 hours. At this time there are five strong updrafts, each possessing greatly enhanced cyclonic vorticity (lower right panel), so that the initial monopole structure of the initial vortex is completely dwarfed by the local vorticity of the VHTs.

Comparing Fig. 8 with Fig. 9 shows that the VHTs move inwards over time. Spiral inertia-gravity waves propagating outwards occur throughout the vortex evolution and these are especially prominent in animations of the vertical velocity fields. The pattern of strong updrafts changes continuously with time, but is mainly monopolar, dipolar, or tripolar and always asymmetric. By the end of the rapid intensification period, no more than three VHTs are active around the circulation centre.

The simulated vortex exhibits many realistic features of a mature tropical cyclone (e.g., Willoughby 1995, Kossin et al. 2002, Corbosiero et al. 2006, Marks et al. 2008) with spiral bands of convection surrounding an approximately symmetric eyewall and a central eye that is free of deep convection (Fig. 9, upper panels). In the mature phase, the evolution of the vortex core is characterised by an approximately axisymmetric circulation superimposed on which are:

- small, but finite-amplitude vortex Rossby waves that propagate azimuthally, radially and vertically on the mean potential vorticity gradient of the system scale vortex (Shapiro and Montgomery 1993, Guinn and Schubert 1993, Montgomery and Kallenbach 1997, Möller and Montgomery 2000, Chen and Yau 2001, Wang 2002a, b, Chen et al. 2003, McWilliams et al. 2003, Martinez 2008, Martinez et al. 2011);[25]
- barotropic-baroclinic shear instabilities (Schubert et al. 1999, Nolan and Montgomery 2000);
- eyewall mesovortices (Schubert et al. 1999, Montgomery et al. 2002, Braun et al. 2006); and
- trochoidal 'wobbling' motion of the inner-core region (Nolan and Montgomery 2000, Nolan et al. 2001).

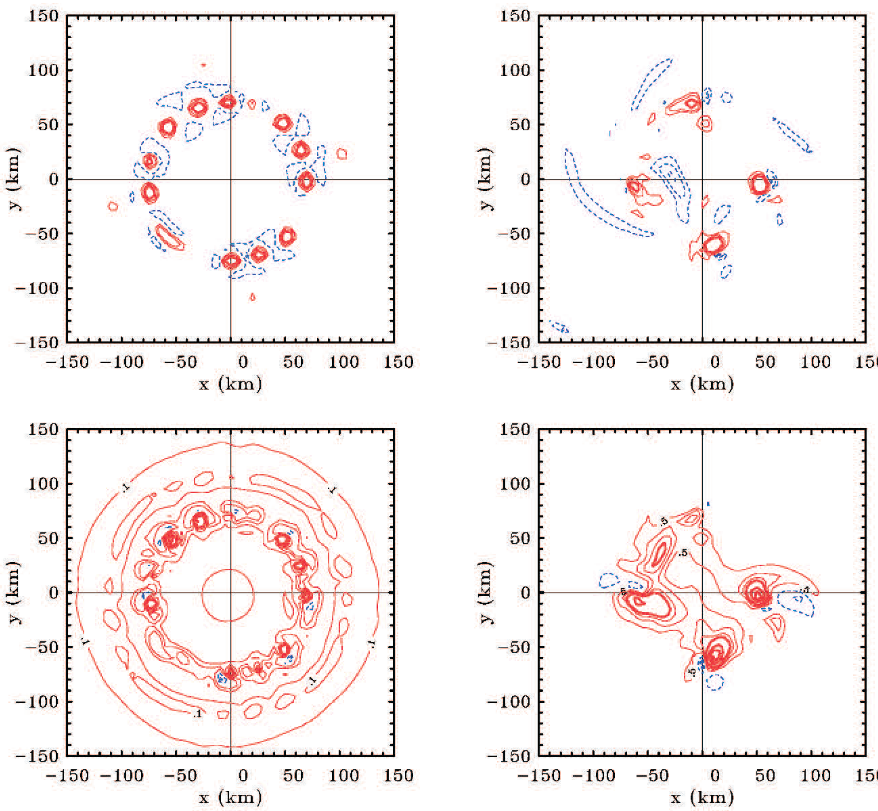
Of course, these processes are not adiabatic and inviscid as they are coupled to the boundary layer and convection (Schecter and Montgomery 2007, Nguyen et al. 2011).

Observational evidence for VHTs

The discovery of VHTs in three-dimensional numerical-model simulations of tropical cyclogenesis and tropical cyclone intensification has motivated efforts to document such structures in observations. Two early studies were those of Reasor et al. (2005), who used airborne Doppler radar data to show that VHTs were present in the genesis phase of Hurricane *Dolly* (1996), and Sippel et al. (2006), who found evidence for VHTs during the development of tropical storm *Alison* (2001). It was not until relatively recently that Houze et al. (2009) presented the first detailed observational evidence of VHTs in a depression that was intensifying and which subsequently became hurricane *Ophelia* (2005). The updraft that they documented was 10 km wide and had vertical velocities reaching 10–25 m s^{-1} in its upper portion, the radar echo of which reached to a height of 17 km. The updraft was contiguous with an extensive stratiform region on the order of 200 km in extent. Maximum values of vertical vorticity averaged over the convective region during different fly-bys were on the order of 5–10 $\times 10^{-4}$ s^{-1} (see Houze et al. 2009, Fig. 20).

[25]The restoring mechanism for vortex Rossby waves and shear instabilities related thereto is associated with the radial and vertical gradient of dry potential vorticity of the system-scale vortex.

Fig. 8. Vertical velocity fields (upper panels) and fields of the vertical component of relative vorticity (lower panels) at 850 hPa at 9.75 hours (left panels) and 24 hours (right panels) in the control experiment in M1. Contour interval for vertical velocity: thick curves 2.0 m s^{-1} and thin solid curves 0.5 m s^{-1} with highest value 1.0 m s^{-1}, thin dashed curves for negative values with interval 0.25 m s^{-1}. Contour interval for relative vorticity: at 9.75 hours, thick curves 5.0 × 10^{-3} s^{-1} and thin curves 1.0 × 10^{-4} s^{-1}; at 24 hours, thick curves 2.0 × 10^{-2} s^{-1} and thin curves 5.0 × 10^{-3} s^{-1}. Positive values are solid curves in red and negative values are dashed curves in blue. The zero contour is not plotted.



Bell and Montgomery (2010) analysed airborne Doppler radar observations from the Tropical Cyclone Structure 2008 field campaign in the western North Pacific and found the presence of deep, buoyant and vortical convective plumes within a vertically-sheared, westward-moving pre-depression disturbance that later developed into typhoon *Hagupit*. Raymond and Lopez-Carillo (2011) carried out a similar analysis of data from the same field experiment, in their case for different stages during the intensification of typhoon *Nuri* and provided further evidence for the existence of VHT-like structures.

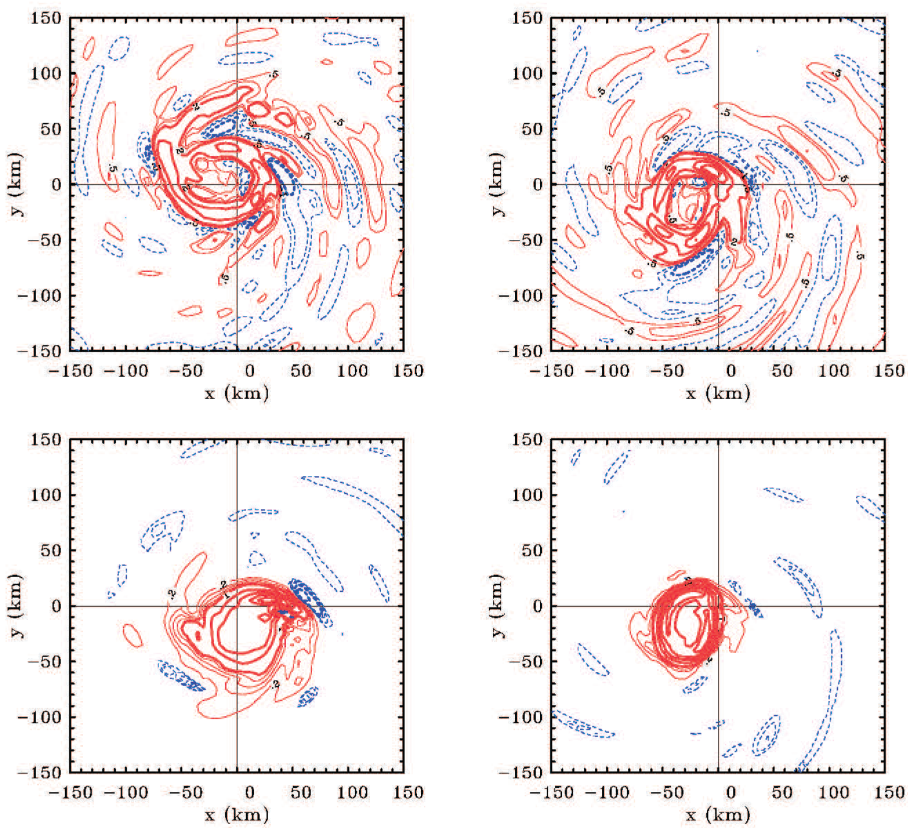
Predictability issues

The association of the inner-core flow asymmetries with the development of VHTs led to the realisation that the asymmetries, like the deep convection itself, would have a limited 'predictability'. The reason is that the pattern of convection is strongly influenced by the low-level moisture field (Nguyen et al. 2008, Shin and Smith 2008). Further, observations have shown that low-level moisture has significant variability on small spatial scales and is not well sampled (Weckwerth 2000), especially in tropical-depression and tropical cyclone environments.

The sensitivity of the flow asymmetries to low-level moisture in an intensifying model cyclone is demonstrated in Nguyen et al. (2008) in an ensemble of ten experiments in which a small (±0.5 g kg^{-1}) random moisture perturbation is added at every horizontal grid point in the innermost domain of the model below 900 mb. As shown in Fig. 10, the evolution in the local intensity as characterised by V_{Tmax} at 900 hPa is similar in all ensemble members, but there is a non-negligible spread, the maximum difference at any given time in the whole simulation being as high as 20 m s^{-1}.

The pattern of evolution of the flow asymmetries is significantly different between ensemble members also. These differences are exemplified by the relative vorticity fields of the control experiment and two randomly-chosen realisations at 24 hours (compare the two panels in Fig. 11 with Fig. 8b). In general, inspection of the relative vorticity and vertical velocity fields of each ensemble member shows similar characteristics to those in the control experiment, the field being non-axisymmetric and still

Fig. 9. Vertical velocity fields (upper panels) and fields of the vertical component of relative vorticity (lower panels) at 850 hPa at 48 h (left panels) and 96 h (right panels) in the control experiment in M1. Contour interval for vertical velocity: thick curves 4.0 m s^{-1} with lowest absolute value 2.0 m s^{-1} and thin curves 0.5 m s^{-1} with highest absolute value 1.0 m s^{-1}. Contour interval for relative vorticity: thick curves 1.0 × 10^{-2} s^{-1} and thin curves 1.0 × 10^{-3} s^{-1}. Positive values are solid curves in red and negative values are dashed curves in blue. The zero contour is not plotted.



dominated by locally intense cyclonic updrafts. However, the detailed pattern of these updrafts is significantly different between the ensemble members.

The foregoing results are supported by the calculations of Shin and Smith (2008), who performed similar ensemble calculations using the minimal three-layer hydrostatic model of Zhu and Smith (2003). Their calculations had twice the horizontal grid spacing used in Nguyen et al. (2008), and a much coarser vertical grid spacing. In fact, the calculations indicate a much larger variance between the ensembles than those in Nguyen et al., suggesting that the predictability limits may be resolution dependent also.

Since the flow on the convective scales exhibits a degree of randomness, t*he convective scale asymmetries are intrinsically chaotic and unpredictable. Only the asymmetric features that survive in an ensemble average of many realisations can be regarded as robust*.

Inclusion of warm-rain physics

The simple representation of condensation in the Nguyen et al. (2008) calculations neglects one potentially important process in tropical cyclone intensification, namely the effects of evaporatively-cooled downdrafts. When these are included through a representation of warm rain processes (i.e. condensation-coalescence without ice), the vortex intensification is delayed and the vortex intensifies more

Fig. 10. Time-series of maximum total wind speed at 900 hPa in the control experiment in Nguyen et al. (2008) (blue) and in the 10 ensemble experiments therein (thin, red).

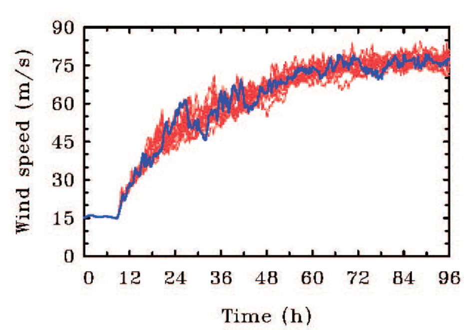


Fig. 11. Relative vorticity fields of 2 representative realisations from the f-plane ensemble at 24 h in Nguyen et al. (2008). Contour interval: (thin curves) 2.0×10^{-3} s^{-1} up to 8.0×10^{-3} s^{-1} and for larger values (thick curves) 1.0×10^{-2} s^{-1}. Positive values are solid curves in red and negative values are dashed curves in blue. The zero contour is not plotted.

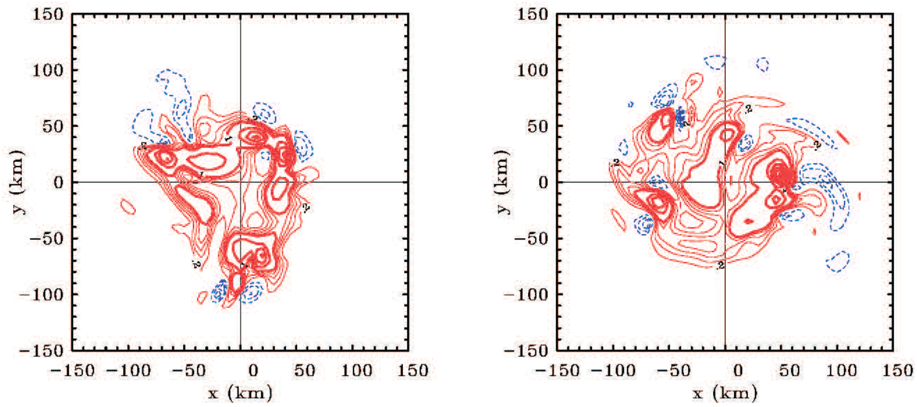
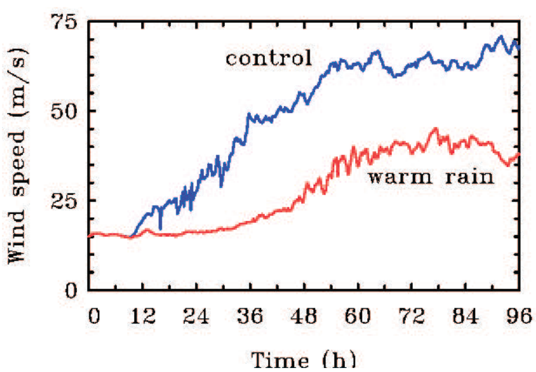


Fig. 12. Time-series of azimuthal-mean maximum tangential-wind speed at 900 hPa in the control experiment in Nguyen et al. (2008) and in the corresponding experiment with a representation of warm rain processes.



slowly than in the control experiment (see, e.g. Fig. 12). The intensity after four days is considerably lower also. Nguyen et al. (2008) attributed this lower intensity to a reduction in the convective instability that results from downdrafts associated with the rain process and to the reduced buoyancy in clouds on account of water loading.

Summary of the new paradigm

The foregoing simulations, both on an f-plane and on a β-plane, show that the VHTs dominate the intensification period at early times and that the progressive aggregation, merger and axisymmetrization of the VHTs together with the low-level convergence they generate are prominent features of the tropical cyclone intensification process.

The emerging flow asymmetries are not merely a manifestation of the numerical representation of the equations,[26] but rather a reflection of a key physical process, namely moist convective instability. The results of Nguyen et al. support those of Montgomery et al. (2006) in an idealised study of the transformation of a relatively-weak, axisymmetric, middle-level, cold-cored vortex on an f-plane to a warm-cored vortex of tropical cyclone strength and by those of near cloud-resolving numerical simulations of tropical storm *Diana* (1984) (Davis and Bosart 2002; Hendricks et al. 2004), which drew attention to the important role of convectively-amplified vorticity in the spin-up process. In turn, the results are supported by more recent calculations of the intensification process using a range of different cloud-representing models with a refined horizontal resolution (Fang and Zhang 2011, Gopalakrishnan et al. 2011, Bao et al. 2012, Persing et al. 2013). All of these studies indicate that VHTs are the basic coherent structures of the tropical cyclone intensification process.

In the Nguyen et al. (2008) simulations, axisymmetrization is not complete after 96 h and there is always a prominent low azimuthal-wave-number asymmetry (often with wave number 1 or 2) in the inner-core relative vorticity. Therefore the question arises: *to what extent are the axisymmetric paradigms discussed in the two earlier sections of this article relevant to explaining the spin-up in the three-dimensional model? In particular, do they apply to the azimuthally -averaged fields in the model?* We address this question in the following section.

[26]It is true that the initial convective updrafts in the control experiment in Nguyen et al. (2008) form more or less within an annular envelope inside the radius of maximum tangential winds and that their precise location, where grid-scale saturation occurs first in this annulus, is determined by local asymmetries associated with the representation of an initially symmetric flow on a square grid. However, the updrafts rapidly forget their initial configuration and become randomly distributed.

An axisymmetric view of the new paradigm

The new paradigm for tropical cyclone spin-up discussed in the previous section highlights the collective role of rotating deep convection in amplifying the storm circulation. These convective structures are intrinsically asymmetric, but it remains possible that the cooperative intensification theory, the WISHE theory, or a modification of these may still provide a useful integrated view of the intensification process in an azimuthally-averaged sense. With this possibility in mind, Smith et al. (2009) examined the azimuthally-averaged wind fields in the control experiment of Nguyen et al. (2008) and found that they could be interpreted in terms of a version of the cooperative intensification theory with an important modification. This modification, discussed below, recognizes that as intensification proceeds, the spin-up becomes progressively focussed in the boundary layer.

Until recently, the perception seems to have been that surface friction plays only an inhibiting role in vortex intensification. For example, Raymond et al. (1998, 2007) assume that the boundary layer is generally responsible for spin-down. Specifically, Raymond et al. (2007) note that '…cyclone development occurs when the tendency of convergence to enhance the low-level circulation of a system defeats the tendency of surface friction to spin the system down', while Marin et al. (2009) state that: 'The primary balance governing the circulation in the planetary boundary layer is between the convergence of environmental vorticity, which tends to spin-up the storm, and surface friction, which tends to spin it down.'

A similar idea is presented in Kepert's (2011) review on the role of the boundary-layer circulation: 'The boundary-layer circulation can only *spin down* (our emphasis) the storm, since inwards advection of (absolute, our insertion) angular momentum by the frictional inflow is countered by the surface frictional torque and by outwards advection in the return branch of the gyre.' We find this statement misleading. Indeed, long ago, Anthes (1971) discussed '… the paradox of the dual role of surface friction, with increased friction yielding more intense circulation…'. This idea of a dual role is different from that envisaged by Charney and Eliassen (1964) in that it focuses on the dynamical role of the boundary layer as opposed to its thermodynamical role in supplying moisture to feed the inner-core convection. In retrospect, Anthes' statement points to the possibility that the spin-up of the inner-core winds might actually occur within the boundary layer. This possibility is further supported by the ubiquitous tendency in numerical models of the vortex boundary layer for supergradient winds to develop in the layer (e.g. Zhang et al. 2001, Kepert and Wang 2001, Nguyen et al. 2002, Smith and Vogl 2008).

There was already some evidence that spin-up occurs in the boundary layer in unpublished calculations performed by our late colleague, Wolfgang Ulrich. Using a simple axisymmetric tropical cyclone model and performing back trajectory calculations, he found that in all calculations examined, the ring of air associated with the maximum tangential wind speed invariably emanated from the boundary layer at some large radius from the storm axis. Numerical simulations of hurricane *Andrew* (1992) by Zhang et al. (2001) indicated that spin-up occurred in the boundary layer of this storm.

Two spin-up mechanisms

Smith et al.'s (2009) analysis indicated the existence of two mechanisms for the spin-up of the azimuthal-mean tangential circulation of a tropical cyclone, both involving the radial convergence of M (defined in section 2.4). The first mechanism is associated with the radial convergence of *M above the boundary layer* induced by the inner-core convection as discussed earlier in this article. It explains why the vortex expands in size and may be interpreted in terms of balance dynamics (see section 'An axisymmetric balance view of spin-up' later in this article).

The second mechanism is associated with the radial convergence of *M within the boundary layer* and becomes progressively important in the inner-core region as the vortex intensifies. Although M is *not* materially conserved in the boundary layer, the largest wind speeds anywhere in the vortex can be achieved in the boundary layer. This happens if the radial inflow is sufficiently large to bring the air parcels to small radii with a minimal loss of M. Stated in another way, the reduction of M in the formula for v is more than offset by the reduction in r.[27] This mechanism is coupled to the first through boundary-layer dynamics because in boundary-layer theory the radial pressure gradient of the boundary layer is slaved to that of the interior flow as discussed earlier.[28]

The spin-up of the inner-core boundary layer requires the radial pressure gradient to increase with time, which, in turn, requires spin-up of the tangential wind at the top of the boundary layer by the first mechanism. The boundary layer spin-up mechanism explains why the maximum azimuthally-averaged tangential wind speeds are located near the top of the boundary layer in the model calculations of Smith et al. (2009) and others (e.g. Braun and Tao 2000, Zhang et al. 2001, Bao et al. 2012, Gopalakrishnan et al. 2012), and in observations (Kepert 2006a, b, Montgomery et al. 2006, Bell and Montgomery 2008, Schwendike and Kepert 2008, Sanger et al. 2014, Zhang et al. 2011a).

The second spin-up mechanism is not unique to tropical cyclones, but appears to be a feature of other rapidly-

[27]As discussed earlier in this article, an alternative, but equivalent interpretation for the material rate of change of v follows directly from Newton's second law in which, neglecting eddy processes, the driving force is the generalised Coriolis force (see footnote 6). If there is inflow, this force is positive for a cyclonic vortex and this term will contribute to the material increase of v. If rings of air can converge quickly enough (i.e. if $|\mathbf{u}|$ is sufficiently large), the generalised Coriolis force can exceed the tangential component of frictional force and the tangential winds of air parcels will increase with decreasing radius in the boundary layer. It is precisely for this reason that supergradient winds can arise in the boundary layer.

[28] In Smith et al. (2009) these spin-up processes were erroneously stated to be independent.

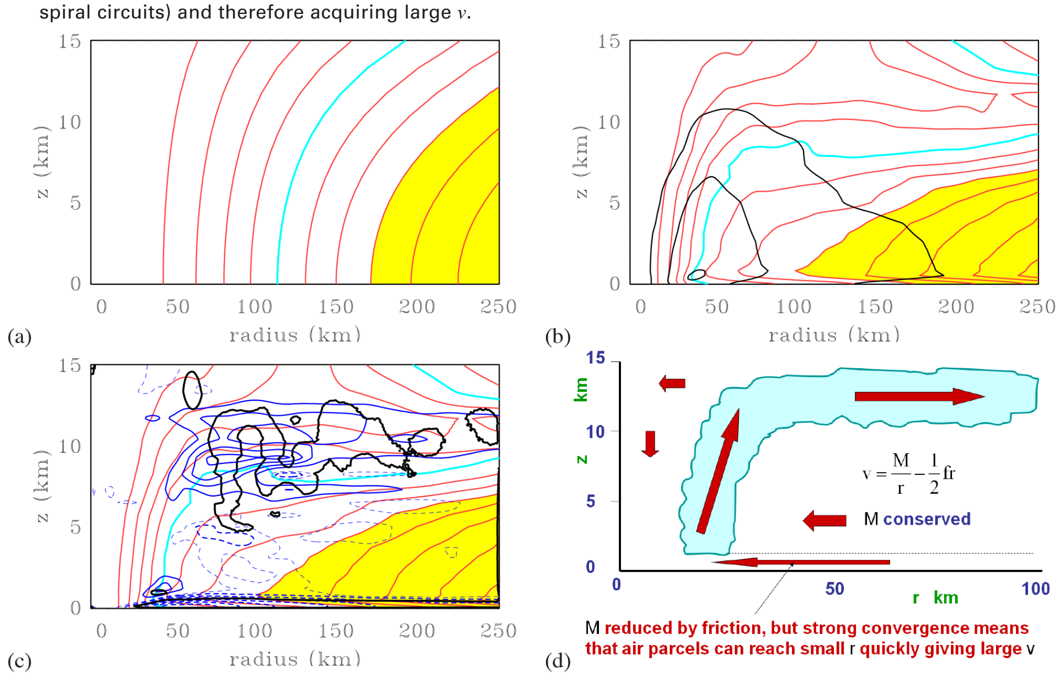
rotating atmospheric vortices such as tornadoes, waterspouts and dust devils in certain parameter regimes, and is manifest as a type of axisymmetric vortex breakdown (Smith and Leslie 1976, Howells et al. 1988, Lewellyn and Lewellyn 2007a,b).

The two mechanisms may be illustrated succinctly by the evolution of the azimuthally-averaged M surfaces from the control experiment in Nguyen et al. (2008) shown here in Fig. 13. The left panel depicts the M surfaces at the initial time of model integration. One M surface is coloured in cyan and those larger than a certain value are shaded in yellow to highlight their inward movement with time. We note that the initial vortex is centrifugally stable so that the M surfaces increase monotonically outwards. Panel (b) shows the M surfaces at 48 hours integration time together with three contours of the tangential wind component with values 17 m s^{-1} (gale force strength), 34 m s^{-1} (hurricane force) and 51 m s^{-1}. Panel (c) is a repeat of panel (b), but with the radial wind plotted in blue instead of the tangential component. Shown also is the zero contour of the discriminant D defined in Eqn. (13). Panel (d) presents a schematic of the revised dynamical view of tropical cyclone intensification. The boundary layer as defined here is the layer of strong radial wind associated with the effect of surface friction. This layer is about 1 km deep.

The inflow velocities above the boundary layer are typically less than 1 m s^{-1} through the lower and middle troposphere. Nevertheless, this weak inflow is persistent, carrying the M surfaces inwards and accounting for the amplification of the tangential winds above the boundary layer. Across the boundary layer, the M surfaces have a large inward slope with height reflecting the removal of M near the surface by friction. Notably, the largest inward displacement of the M surfaces occurs near the top of the boundary layer where frictional stresses are small. This pattern of M surfaces explains why the largest tangential wind speed at a given radius occurs near the top of the boundary layer. Note that the bulge of the highlighted cyan contour of M is approximately where the tangential wind speed maximum occurs and that this contour has moved radially inwards about 60 km by 48 h.

There are two regions of strong outflow. The first is just above the inflow layer in the inner-core region, where the tangential velocity is a maximum and typically supergradient. The second is in the upper troposphere and marks the outflow layer produced by the inner-core convection. As air parcels move outwards, M is approximately conserved so that the M surfaces are close to horizontal. However, beyond a radius of about 100 km, the M surfaces slope downwards, whereupon the radial gradient of M is negative.

Fig. 13. Absolute angular momentum surfaces at (a) the initial time, and (b) at 48 hours (red and pink contours). Contour interval 4×10^5 m^2 s^{-1}. Thick cyan contour has the value 20×10^5 m^2 s^{-1}. The region shaded in yellow indicates M values $\geq 32 \times 10^5$ m^2 s^{-1}. Shown also in (b) are contours of the tangential wind component with values 17 m s^{-1} (gale force strength), 34 m s^{-1} (hurricane force) and 51 m s^{-1}; (c) is a repeat of panel (b), but with contours of the radial wind plotted in blue instead of the tangential component. Contour interval for u is 2 m s^{-1} (the thin dashed contour shows the 1 m s^{-1} inflow). The black contours delineate the zero value of the discriminant D in Eqn. (13) and encircle regions in which the azimuthally-averaged flow satisfies the necessary condition for symmetric instability. (d) Schematic of the revised dynamical view of tropical cyclone intensification. The only difference between this figure and Fig. 4 is the highlighted dynamical role of the boundary layer, wherein air parcels may reach small radii quickly (minimising the loss of M during spiral circuits) and therefore acquiring large v.



Consequently, regions develop in which the discriminant D becomes negative. There is a region also in the boundary layer where D is negative on account of the strong vertical shear of the tangential wind (last term in Eqn. 13).

The foregoing features are found in all of the numerical simulations we have conducted and lead to a revised axisymmetric view of spin-up that includes the cooperative intensification mechanism, but emphasises the important *dynamical role* of the boundary layer (as illustrated in Fig. 13(d)).

The revised view stimulates a number of important questions:

- To what extent can the spin-up processes be captured in a balance dynamics framework?
- What are the differences between spin-up in a three-dimensional model and that in an axisymmetric model?
- How important is the WISHE paradigm to vortex spin-up in three dimensions?
- How sensitive are the spin-up and inner-core flow structure to the parameterization of the boundary layer in a tropical cyclone forecast model?

The first two questions are addressed in the following two subsections and the others in 'Further properties of the rotating convective updraft paradigm' later in this article.

An axisymmetric balance view of spin-up

The applicability of the axisymmetric balance theory summarised earlier, to the revised view of tropical cyclone intensification encapsulated in Fig. 13d was investigated in Bui et al. (2009). That study built upon those of Hendricks et al. (2004) and Montgomery et al. (2006), who used a form of the SE-equation to investigate the extent to which vortex evolution in a three-dimensional, cloud-resolving, numerical model of hurricane genesis could be interpreted in terms of axisymmetric balance dynamics. The idea was to diagnose the diabatic heating and azimuthal momentum sources from azimuthal averages of the model output at selected times and to solve the SE-equation for the balanced streamfunction with these as forcing functions (see section 2.3). Then one can compare the azimuthal-mean radial and vertical velocity fields from the numerical model with those derived from the balanced streamfunction. Hendricks et al. and Montgomery et al. found good agreement between the two measures of the azimuthal-mean secondary circulation and concluded that the vortex evolution proceeded in a broad sense as a balanced response to the azimuthal-mean forcing by the VHTs.

Bui et al. (2009) applied the same methodology to the Nguyen et al. (2008) calculations on an f-plane, described earlier. A specific aim was to determine the separate contributions of diabatic heating and boundary-layer friction to producing convergence of absolute angular momentum above and within the boundary layer, which, as discussed in 'Two spin-up mechanisms', are the two intrinsic mechanisms of spin-up in an axisymmetric framework. The study investigated also the extent to which a balance approach is useful in understanding vortex evolution in the Nguyen et al. calculations.

The development of regions of symmetric instability in the upper-tropospheric outflow layer and the frictional inflow layer in the Nguyen et al. calculations generally requires the regularization of the SE-equation prior to its solution. A consequence of symmetric instability is the appearance of a shallow layer (or layers) of inflow or reduced outflow in the upper troposphere, both in the three-dimensional model and in the non-regularised balanced solutions, which were examined also by Bui et al. (2009). Using this regularisation procedure it was found that, during the intensification process, the balance calculation captures a major fraction of the azimuthally-averaged secondary circulation, except in the boundary layer where the gradient wind balance assumption breaks down (see Bui et al. Fig. 5 and Fig. 6). Moreover, the diabatic forcing associated with the inner-core convection negates the divergence above the boundary layer that would be induced by friction alone, producing radial inflow both within the boundary layer and in the lower troposphere (Bui et al. 2009, Fig. 4).

An important limitation of the balanced solution is that it considerably underestimates both the strength of the inflow and the intensification rate in the boundary layer, a result that reflects its inability to capture the important inertial effects of the boundary layer in the inner core region (Bui et al. 2009, Figs. 5, 6 and 9). Specifically, these effects include the development of supergradient winds in the boundary layer, the ensuing rapid radial deceleration of the inflow and the eruption of the boundary layer into the interior. Some consequences of these important inertial effects are discussed in later sections of this article.

Comparison of spin-up in axisymmetric and three-dimensional models

The distinctly asymmetric processes of evolution found in the three-dimensional model simulations naturally raises the question: *how does the intensification process compare with that in an axisymmetric model*? Recently, Persing et al. (2013) examined the differences between tropical cyclone evolution in three-dimensional and axisymmetric configurations for the prototype intensification problem described in the Introduction. The study identified a number of important differences between the two configurations. Many of these differences may be attributed to the dissimilarity of deep cumulus convection in the two models. For example, there are fundamental differences in convective organisation.

Deep convection in the three-dimensional model is tangentially-sheared by the differential angular rotation of the azimuthally-averaged system-scale circulation in the radial and vertical directions, unlike convective rings in the axisymmetric configuration. Because convection is not organized into concentric rings during the spin-up process, the azimuthally-averaged heating rate is

considerably less than that in the axisymmetric model. For most of the time this lack of organisation results in slower spin-up and leads ultimately to a weaker mature vortex. There is a short period of time, however, when the rate of spin-up in the three-dimensional model exceeds that of the maximum spin-up rate in the axisymmetric one. During this period the convection is locally more intense than in the axisymmetric model and the convection is organised in a quasi ring-like structure resembling a developing eyewall. These regions of relatively strong updrafts have an associated vertical eddy momentum flux that contribute significantly to the spin-up of the azimuthal mean vortex and provide an explanation for the enhanced spin-up rate in the three-dimensional model despite the relative weakness of the azimuthal mean heating rate in this model.

Consistent with findings of previous work (Yang et al. 2007), the mature intensity in the three-dimensional model is reduced relative to that in the axisymmetric model. In contrast with previous interpretations invoking barotropic instability and related mixing processes as a mechanism detrimental to the spin-up process, the results suggest that eddy processes associated with vertical plume structures can assist the intensification process via up-gradient momentum fluxes in the radial direction. These plumes contribute also to the azimuthally-averaged heating rate and the corresponding azimuthal-mean overturning circulation. Persing et al.'s analysis has unveiled a potentially important issue in the representation of subgrid scale parameterisations of eddy momentum fluxes in hurricane models. Comparisons between the two model configurations indicate that the structure of the resolved eddy momentum fluxes above the boundary layer differs from that prescribed by the subgrid-scale parameterisations in either the three-dimensional or axisymmetric configurations, with the exception of the resolved horizontal eddy momentum flux during the mature stages.

Another important difference between the two configurations is that the flow fields in the axisymmetric model tend to be much noisier than in the three- dimensional model. The larger flow variability is because the deep convection generates azimuthally-coherent, large-amplitude, inertia-gravity waves. Although deep convection in the three-dimensional model generates inertia-gravity waves also, the convection is typically confined to small ranges of azimuth and tends to be strained by the azimuthal shear. These effects lead to a reduced amplitude of variability in the azimuthally-averaged flow fields.

Further properties of the rotating convective updraft paradigm

Is WISHE essential?

To explore the relationship of the new intensification paradigm to that of the evaporation-wind feedback mechanism discussed earlier in this article, Nguyen et al. (2008) conducted a preliminary investigation of the intensification of the vortex when the dependence of wind speed in the bulk aerodynamic formulae for latent- and sensible-heat fluxes was capped at 10 m s^{-1}. This wind-speed cap gave sea-to-air water vapour fluxes that never exceeded 130 W m^{-2}, a value comparable to observed trade-wind values in the summer-hemisphere.

A more in depth study of the role of surface fluxes was carried out by Montgomery et al. (2009) and the principal results are summarized in their Fig. 9. When the wind-speed dependence of the surface heat fluxes is capped at 10 m s^{-1}, the maximum horizontal wind speed is a little less than in the uncapped experiments, but the characteristics of the vortex evolution are qualitatively similar, in both the pseudo-adiabatic experiments and when warm rain processes are included. In contrast, when the latent and sensible heat fluxes are suppressed altogether, the system-scale vortex does not intensify, even though there is some transient hot-tower convection and local wind-speed enhancement. Despite the nonzero CAPE present in the initial sounding, no system-scale amplification of the wind occurs. This finding rules out the possibility that vortex intensification with capped fluxes is an artefact of using an initially unstable moist atmosphere (cf. Rotunno and Emanuel, 1987). A minimum value of approximately 3 m s^{-1} was found necessary for the wind- speed cap at which the vortex can still intensify to hurricane strength in the pseudo-adiabatic configuration discussed above.

When warm rain processes are included, the vortex still intensifies with a wind-speed cap of 5 m s^{-1}. These experiments suggest that large latent heat fluxes in the core region are not necessary for cyclone intensification to hurricane strength and indicate that the evaporation-wind feedback process discussed earlier is not essential.

A word of caution is needed if one tries to interpret the intensification process discussed in the earlier section 'A new rotating-convective updraft paradigm' in terms of the schematic of Fig. 6. At first sight it might appear that a capped wind-speed in the sea-to-air water vapour flux no longer gives the needed boost in the inner-core boundary layer mixing ratio and concomitant elevation of θ_e required for an air parcel to ascend the warmed troposphere created by prior convective events. The pitfall with this idea is that *it is not necessary to have the water vapour flux increase with wind speed to generate an increase in boundary layer moisture and hence an increase in boundary layer θ_e. The boundary layer θ_e* will continue to rise as long as the near-surface air remains unsaturated at the sea surface temperature. In some sense, this discussion echoes what was said earlier in this article

concerning the increase in saturation mixing ratio with decreasing surface pressure. However, the above results add further clarification by showing that the wind-speed dependence of the surface moisture flux is not essential for sustained intensification, even though it may generally augment the intensification process.[29]

Dependence of spin-up on the boundary-layer parameterisation

A recent assessment of the state-of-the-art Advanced Hurricane Weather Research and Forecasting model (WRF; Skamarock et al. 2005) by Davis et al. (2007) examined, inter alia, the sensitivity of predictions of hurricane *Katrina* (2005) to the model resolution and the formulation of surface momentum exchange. Interestingly, it did not consider that there might be a sensitivity also to the boundary-layer parameterisation used in the model. This omission reflects our experience in talking with model developers that the choice of boundary-layer parameterisation is low on the list of priorities when designing models for the prediction of tropical cyclones: in several instances the modeller had to go away and check exactly which scheme was in use! In view of this situation, and in the light of the theoretical results described above indicating the important role of boundary-layer dynamics and thermodynamics on tropical cyclone spin-up, we review briefly some work that has attempted to address this issue. Three important questions that arise are: how sensitive are the predictions of intensification and structure to the choice of boundary-layer parameterisation scheme; how robust are the findngs of Nguyen et al. (2008), Montgomery et al. (2009) and Smith et al. (2009) to the choice of boundary-layer scheme; and which scheme is optimum for operational intensity prediction? These questions have been addressed to some extent in the context of case studies (Braun and Tao 2000, Nolan et al. 2009a,b) and idealised simulations (Hill and Lackmann 2009, Smith and Thomsen 2010, Bao et al. 2012, Gopalakrishnan et al. 2013), but the question concerning the optimum scheme remains to be answered.

Smith and Thomsen (2010) investigated tropical cyclone intensification in an idealised configuration using five different boundary-layer parameterisation schemes available in the MM5 model, but with all the schemes having the same surface exchange coefficients. Values of these coefficients were guided by results from the coupled boundary-layer air-sea transfer experiment[30] (CBLAST) to facilitate a proper comparison of the schemes. Like Braun and Tao, they found a significant sensitivity of vortex evolution, final intensity, inner-core low-level wind structure, and spatial distribution of vertical eddy diffusivity, K, to the particular scheme used. In particular, the less diffusive schemes have shallower boundary layers and more intense radial inflow, consistent with some early calculations of Anthes (1971) using a constant-K formulation. The stronger inflow occurs because approximately the same surface stress is distributed across a shallower layer leading to a larger inward agradient force in the outer region of the vortex. In turn, the stronger inflow leads to stronger supergradient flow in the inner core.

The sensitivity to K is problematic because the only observational estimates for this quantity that we are aware of are those analysed recently from flight-level wind measurements at an altitude of about 500 m in hurricanes *Allen* (1980) and *Hugo* (1989) by Zhang et al. (2011b). In *Hugo*, maximum K-values were about 110 m^2 s^{-1} beneath the eyewall, where the near-surface wind speeds were about 60 m s^{-1}, and in Allen they were up to 74 m^2 s^{-1}, where wind speeds were about 72 m s^{-1}. Based on these estimates. one would be tempted to judge that the MRF and Gayno-Seaman schemes studied by Braun and Tao and Thomsen and Smith are much too diffusive as they have maximum values of K on the order of 600 m^2 s^{-1} and 250 m^2 s^{-1} respectively. On the other hand, the other schemes have broadly realistic diffusivities.

From the latter results alone, it would be premature to draw firm conclusions from a comparison with only two observational estimates.[31] However, in recent work, Zhang et al. (2011a) have analysed the boundary layer structure of a large number of hurricanes using the Global Positioning System dropwindsondes released from National Oceanic and Atmospheric Administration and United States Air Force reconnaissance aircraft in the Atlantic basin. This work is complemented by individual case studies of hurricanes and typhoons examining the boundary-layer wind structure of the inner-core region (Kepert 2006a, b; Montgomery et al. 2006; Bell and Montgomery 2008; Schwendike and Kepert 2008; Sanger et al. 2014). These results show that the inner-core boundary-layer depth is between 500 m and 1 km.[32] If the observed boundary layer depth is taken to be an indication of the level of diffusivity, these results imply that boundary layer schemes with deep inflow layers, such as the MRF scheme, are too diffusive and fail to capture the important inertial dynamical processes discussed in the earlier subsection 'Two spin-up mechanisms'.

---

[29]These considerations transcend the issue of axisymmetric versus three-dimensional intensification. While it has been shown that tropical cyclones can intensify without evaporation-wind feedback in an axisymmetric configuration (Montgomery et al. 2009, their section 5.3), our focus here is on the three-dimensional problem, which we believe to be the proper benchmark.

[30]See Black et al. (2007), Drennan et al. (2007), French et al. (2007), Zhang et al. (2008).

[31]Since this paper was first drafted, Zhang and Montgomery (2012) obtained values of vertical diffusivity for Category 5 hurricane *David* (1979) that are comparable to these values and obtained estimates of horizontal diffusivity for hurricanes *Hugo* (1989), *Allen* (1980) and *David* (1979) in the boundary layer also. An additional paper by Zhang and Drennan (2012) used the CBLAST data in the rainband region of the hurricanes *Fabian* (2003), *Isabel* (2003), *Frances* (2004) and *Jeanne* (2004) to obtain vertical profiles of the vertical diffusivity with comparable, but somewhat weaker values to the values found by Zhang et al. (2011). In summary, we now have estimates of vertical diffusivity from seven different storms.

[32]These studies show also that the maximum mean tangential velocity occurs within the boundary layer, supporting the revised conceptual model presented in the subsection 'Two spin-up mechanisms'.

The idealised numerical calculations of Nguyen et al. (2008) described earlier in this article employed a relatively simple bulk boundary-layer parameterisation scheme, albeit a more sophisticated one than the slab model used by E97. For this reason, the calculations were repeated by Smith and Thomsen (2010) using a range of boundary- layer schemes having various degrees of sophistication. While the latter study showed quantitative differences in the intensification rate, mature intensity and certain flow features in the boundary layer for different schemes, in all cases the maximum tangential wind was found to occur within, but close to the top of the strong inflow layer (see Smith and Thomsen, Fig. 2), implying that the boundary-layer spin-up mechanism articulated in the subsection 'Two spin-up mechanisms' is robust.

The studies by Braun and Tao and Thomsen and Smith have elevated awareness of an important problem in the design of deterministic forecast models for hurricane intensity, namely which boundary layer scheme is most appropriate? They provide estimates also of forecast uncertainty that follow from the uncertainty in not knowing the optimum boundary-layer scheme to use. In an effort to address this issue, Kepert (2012) compared a range of boundary-layer parameterisation schemes in the framework of a steady-state boundary-layer model in which the tangential wind speed at the top of the boundary layer is prescribed and assumed to be in gradient wind balance. As a result of his analyses, he argues that boundary-layer schemes that do not reproduce the near- surface logarithmic layer should not be used. However, Smith and Montgomery (2013) present both observational and theoretical evidence that calls into question the existence of a near-surface logarithmic layer in the inner core of a tropical cyclone.

Conclusions

In his recent review of tropical cyclone dynamics, Kepert (2011) states that 'the fundamental view of tropical cyclone intensification as a cooperative process between the primary and secondary circulations has now stood for four decades, and continues to underpin our understanding of tropical cyclones'. Here we have provided what we think is a broader perspective of the basic intensification process including a new rotating convective updraft paradigm and a boundary-layer spin-up mechanism not discussed by Kepert.

In the early days of hurricane research, the intensification problem was viewed and modelled as an axisymmetric phenomenon and, over the years, three main paradigms for intensification emerged: the CISK paradigm; the cooperative intensification paradigm; and the WISHE paradigm. We have analysed these paradigms in detail and have sought to articulate the relationship between them. We noted that the process of spin-up is similar in all of them, involving the convectively-induced inflow in the lower troposphere, which draws in absolute angular momentum surfaces to amplify the tangential wind component. This process is less transparent in the WISHE paradigm, which is usually framed using potential radius coordinates. The main processes distinguishing the various paradigms are the methods for parameterising deep convection and, in the case of the CISK paradigm, the omission of explicit surface moisture fluxes in maintaining the boundary-layer moisture and the reliance on ambient CAPE.

The new paradigm is distinguished from those above by recognising the presence of localised, rotating deep convection that grows in the rotation-rich environment of the incipient storm, thereby greatly amplifying the local vorticity. As in the axisymmetric paradigms, this convection collectively draws air inwards towards the circulation axis and, in an azimuthally-averaged sense, draws in absolute angular momentum surfaces in the lower troposphere. However, unlike the strictly axisymmetric configuration, deep convection in the three-dimensional model is tangentially-sheared by the differential angular rotation of the system-scale circulation in the radial and vertical directions. Because convection is not generally organised into concentric rings during the spin-up process, the azimuthally-averaged heating rate and the corresponding radial gradient thereof are considerably less than that in the corresponding axisymmetric configuration. For most of the time this lack of organisation results in slower spin-up and leads ultimately to a weaker mature vortex. In contrast with previous interpretations invoking barotropic instability and related mixing processes as a mechanism detrimental to the spin-up process, these vortical plume structures can assist the intensification process via up-gradient momentum fluxes.

In the new paradigm, the spin-up process possesses a stochastic component, which reflects the convective nature of the inner-core region. From a vorticity perspective, it is the progressive aggregation and axisymmetrisation of the vortical convective plumes that build the system-scale vortex core, but axisymmetrization is never complete. There is always a prominent low azimuthal-wave-number asymmetry in the inner-core relative vorticity and other fields. In the mature phase, the evolution of the vortex core is characterised by an approximately axisymmetric circulation superimposed on which are primarily small, but finite-amplitude vortex Rossby waves, eyewall mesovortices, and their coupling to the boundary layer and convection.

The axisymmetric (or 'mean field') view that results after azimuthally averaging the three-dimensional fields provides a new perspective into the axisymmetric dynamics of the spin-up process. When starting from a prototypical vortex less than tropical-storm strength, there are two mechanisms for spinning up the mean tangential circulation. The first involves convergence of absolute angular momentum above the boundary layer where this quantity is approximately materially conserved. This mechanism acts to amplify the bulk circulation and is captured in a balance theory wherein the convergence can be thought of as being 'driven' by the radial gradient of diabatic heating associated with deep inner-core convection.

In particular, it acts to broaden the outer circulation.

The second mechanism involves the convergence of absolute angular momentum within the boundary layer, where this quantity is not materially conserved, but where air parcels are displaced much further radially inwards than air parcels above the boundary layer. This mecha- nism is responsible for producing the maximum tangen- tial winds in the boundary layer and for the generation of supergradient wind speeds there. The mechanism is cou- pled to the first through boundary-layer dynamics because, according to boundary layer theory, the radial pressure gradient is slaved to that of the interior flow. The spin-up of the inner-core boundary layer requires the radial pressure gradient to increase with time, which, in turn, requires spin-up of the tangential wind at the top of the boundary layer by the first mechanism. The second spin-up mechanism is not unique to tropical cyclones, but appears to be a feature of other rapidly-rotating atmospheric vortices such as tornadoes, waterspouts and dust devils in certain parameter regimes, and is manifest as a type of axisymmetric vortex breakdown. This mechanism cannot be represented by balance theory.

In the new paradigm, the underlying fluxes of latent heat are necessary to support the intensification of the vortex. However, the wind-evaporation feedback mechanism that has become the accepted paradigm for tropical cyclone intensification is not essential, nor is it the dominant intensification mechanism in the idealised numerical experiments that underpin the paradigm.

The road ahead

The ability to numerically simulate a hurricane in near real-time with high horizontal resolution using complex microphysical parameterisations can lead to a false sense of understanding without commensurate advances in understanding of the underlying fluid dynamics and thermodynamics that support the intensification process. The situation is elegantly summed up by Ian James who, in reference to the Held-Hou model for the Hadley circulation, writes: 'This is not to say that using such [simple] models is folly. Indeed the aim of any scientific modelling is to separate crucial from incidental mechanisms. Comprehensive complexity is no virtue in modelling, but rather an admission of failure.' (James 1994, p93.) We endorse this view whole-heartedly in the context of tropical cyclone modelling.

We believe that a consistent dynamical framework of tropical cyclone intensification is essential to diagnose and offer guidance for improving the three-dimensional hurricane forecast models being employed in countries affected by these deadly storms. To this end, we have provided a review and synthesis of intensification theory for a weak, but finite amplitude, cyclonic vortex in a quiescent tropical atmosphere. However, there is still much work to be done. As intimated in the Introduction, there is a need to investigate systematically the effects of an environmental flow, including one with vertical shear, on the dynamics of the intensification process.

There is a need also to document further the lifecycle of VHTs and the way they interact with one another in tropical-depression environments through analyses of existing field data and those from future field experiments. We see a role also for idealised numerical simulations to complement these observational studies.

There is a need to investigate the limits of predictability of intensity and intensity change arising from the stochastic nature of the VHTs, which has potential implications for the utility of assimilating Doppler radar data as a basis for deterministic intensity forecasts.

Finally, a strategy is required to answer the question: what is the 'optimum' boundary-layer parameterisation for use in numerical models used to forecast tropical cyclone intensity? The current inability to determine 'the optimum scheme' has implications for the predictability of tropical cyclone intensification using current models.

Acknowledgments

This paper was begun during the authors' visit to The Vietnamese National University (VNU) in Hanoi in January 2010 and a first draft was completed in June of the same year during a visit to Taiwan National University in Taipei (NTU). We are grateful to Prof. Phan Van Tan of the VNU and Prof. Chun-Chieh Wu of the NTU for providing us with a stimulating environment for our work. We are grateful also for perceptive reviews of the original manuscript by Kerry Emanuel, Sarah Jones, John Molinari, Dave Raymond and an anonymous reviewer.

MTM acknowledges the support of Grant No. N00014-03-1-0185 from the U.S. Office of Naval Research and from the U.S. Naval Postgraduate School, NOAA's Hurricane Research Division, NSF ATM 0649944, NSF ATM 0715426 and NSF AGS 0733380. RKS acknowledges previous financial support for hurricane research from the German Research Council (Deutsche Forschungsgemeinschaft) under grants SM 30/23 and SM 30/25.

References

Ahrens, C.D. 2008. *Meteorology today: An introduction to weather, climate and the environment*. Thomson Brooks/Cole, Stamford, Connecticut, 624pp.

Anthes, R.A. 1971. Iterative solutions to the steady-state axisymmetric boundary-layer equations under an intense pressure gradient. *Mon. Wea. Rev.*, *99*, 261–68.

Asnani, G.C. 2005. *Tropical meteorology*, revised edition. Pune, India.

Bao J.W., Gopalakrishnan, S.G., Michelson, S.A., Montgomery, M.T., 2012. Impact of physics representations in the HWRFX on simulated hurricane structure and pressure-wind relationships. *Mon. Wea. Rev.*, *140*, 3278–99.

Bell, M.M. and Montgomery, M.T. 2008. Observed structure, evolution, and potential intensity of Category 5 hurricane *Isabel* (2003) from 12 to 14 September. *Mon. Wea. Rev.*, *65*, 2025–46.

Bell, M.M. and Montgomery, M.T. 2010. Sheared deep vortical convection in pre-depression *Hagupit* during TCS08. *Geo. Res. Letters*, *37*, L06802.

Bister, M. and Emanuel, K.A. 1998. Dissipative heating and hurricane intensity. *Meteor. Atmos. Phys., 65*, 233–40.

Black, P.G., Marks, F.D. and Black, R.A. 1986. Supercell structure in tropical cyclones. Preprints, Joint Sessions 23d Conf. on Radar Meteorology and Conf. on Cloud Physics, Vol. 3, Snowmass, CO, 22-26 September 1986, Amer. Meteor. Soc., JP255-JP259.

Black, P.G., D'Asaro, E.A., Sanford, T.B., Drennan, W.M., Zhang, J.A., French, J.R., Niiler, P.P., Terrill, E.J. and Walsh, EJ. 2007. Air-sea exchange in hurricanes: synthesis of observations from the coupled boundary layer air-sea transfer experiment. *Bull. Amer. Meteorol. Soc., 88*, 357-374.

Braun, S.A. and Tao, W-K. 2000. Sensitivity of high-resolution simulations of Hurricane Bob (1991) to planetary boundary layer parameterizations. *Mon. Wea. Rev., 128*, 3941–61.

Braun, S.A., Montgomery, M.T. and Pu Z. 2006. High-resolution simulation of hurricane Bonnie (1998). Part I: The organization of eyewall vertical motion. *J. Atmos. Sci., 63*, 19–42.

Braun, S.A., Montgomery, M.T. Mallen, K.J. and Reasor, P.D. 2010 Simulation and interpretation of the genesis of tropical storm *Gert* (2005) as part of the NASA Tropical Cloud Systems and Processes Experiment. *J. Atmos. Sci., 67*, 999–1025.

Bryan, G.H. and Rotunno, R. 2009. Evaluation of an analytical model for the maximum intensity of tropical cyclones. *J. Atmos. Sci., 66*, 3042–60.

Bui, H.H., Smith, R.K., Montgomery, M.T. and Peng, J. 2009. Balanced and unbalanced aspects of tropical-cyclone intensification. *Q. J. R. Meteorol. Soc., 135*, 1715–31.

Chan, J.C.L. and Williams, R.T. 1987 Analytical and numerical studies of the beta-effect in tropical cyclone motion. Part I: Zero mean flow. *J. Atmos. Sci., 44*, 1257–65.

Charney, J.G. and Eliassen, A. 1964. On the growth of the hurricane depression. *J. Atmos. Sci., 21*, 68-75.

Chen, Y. and Yau M.K. 2001 Spiral bands in a simulated hurricane. Part I: vortex Rossby wave verification. *J. Atmos. Sci., 58*, 2128–45.

Chen, Y., Brunet, G. and Yau, M.K. 2003. Spiral bands in a simulated hurricane. Part II: Wave activity diagnostics. *J. Atmos. Sci., 60* , 1240–56.

Corbosiero, K.L., Molinari, J., Aiyyer, A.R. and Black, M.L. 2006. The structure and evolution of hurricane *Elena* (1985). Part II: Convective asymmetries and evidence for vortex Rossby waves. *Mon. Wea. Rev., 134*, 3073–91.

Craig, G.C. and Gray, S.L. 1996. CISK or WISHE as a mechanism for tropical cyclone intensification. *J. Atmos. Sci., 53*, 3528–40.

Davis, C.A. and Bosart, L.F. 2002. Numerical simulations of the genesis of hurricane *Diana* (1984). Part II: Sensitivity of track and intensity prediction. *Mon. Wea. Rev., 130*, 1100–24.

Davis, C.A., Wang, W., Chen, S.S., Chen, Y., Corbosiero, K., DeMaria, M., Dudhia, J., Holland, G., Klemp, J., Michalakes, J., Reeves, H., Rotunno, R., Snyder, C., Xiao, Q. 2008. Prediction of landfalling hurricanes with the advanced hurricane WRF model, *Mon. Wea. Rev., 136*, 1990–2005.

Dengler, K. and Reeder, M.J. 1997. The effects of convection and baroclinicity on the motion of tropical-cyclone-like vortices. *Q. J. R. Meteorol. Soc., 123*, 699–727.

Drennan, W.M. Zhang, J.A. French, J.R. McCormick, C. and Black, P.G. 2007. Turbulent fluxes in the hurricane boundary layer. Part II: Latent heat fluxes. *J. Atmos. Sci., 64*, 1103–15.

Elsberry, R. and Harr, P. 2008: Tropical cyclone structure (TCS08) Field experiment scientific basis, observational platforms, and strategy. *Asia-Pacific Journal of Atmospheric Sciences, 44*, 1–23.

Emanuel, K.A. 1986. An air-sea interaction theory for tropical cyclones. Part I: Steady state maintenance. *J. Atmos. Sci., 43*, 585-604.

Emanuel, K.A. 1988: The maximum intensity of hurricanes. *J. Atmos. Sci., 45*, 1143–55.

Emanuel, K.A. 1989. The finite amplitude nature of tropical cyclogenesis. *J. Atmos. Sci., 46*, 3431–56.

Emanuel, K.A. 1994. *Atmospheric convection*. Oxford University Press, 580pp.

Emanuel, K.A. 1995. Sensitivity of tropical cyclones to surface exchange coefficients and a revised steady-state model incorporating eye dynamics. *J. Atmos. Sci., 52*, 3969–76.

Emanuel, K.A. 1997. Some aspects of hurricane inner-core dynamics and energetics. *J. Atmos. Sci., 54*, 1014–26.

Emanuel K.A. 2003: Tropical cyclones. *Annu. Rev. Earth Planet. Sci., 31*, 75–104.

Emanuel K.A. 2004. Tropical cyclone energetics and structure, *Atmospheric turbulence and mesoscale meteorology*, E. Fedorovich, R. Rotunno and B. Stevens, editors, Cambridge University Press, pp280.

Emanuel, K.A. 2012. Self-stratification of tropical cyclone outflow. Part II: Implications for storm intensification. *J. Atmos. Sci., 69*, 988–96.

Emanuel, K.A. and Raymond, D.J. (eds.) 1993. The representation of cumulus convection in numerical models. *American Meteorological Society, Meteorological Monograph*, no. 46, 246pp.

Emanual, K.A. and Rotunno, R. 2011. Self-stratification of tropical cyclone outflow. Part I: Implications for storm structure. *J. Atmos. Sci., 68*, 2236–49.

Emanuel, K.A., Neelin, J.D. and Bretherton, C.S. 1994: On large-scale circulations of convecting atmospheres. *Q. J. R. Meteorol. Soc., 120*, 1111–43.

Emanuel, K.A., Speer, K., Rotunno, R., Srivastava, R. and Molina, M. 1995: Hypercanes: A possible link in global extinction scenarios. *J. Geophys. Res., 100*, 13 755-765.

Fang, J. and Zhang, F. 2011. Evolution of multiscale vortices in the development of hurricane *Dolly* (2008). *J. Atmos. Sci., 68*, 103–22.

Fiorino, M. and Elsberry, R. 1989. *Some aspects of vortex structure related to tropical-cyclone motion*. *J. Atmos. Sci., 46*, 975–90.

Flatau, M., Schubert, W.H. and Stevens, D.E. 1994. The role of baroclinic processes in tropical cyclone motion: the influence of vertical tilt. *J. Atmos. Sci., 51*, 2589–01.

French, J.R., Drennan, W.M. Zhang, J.A. and Black, P.G. 2007. Turbulent fluxes in the hurricane boundary layer. Part I: Momentum flux, *J. Atmos. Sci., 63*, 1089–1102.

Frisius, T. 2002. Surface-flux-induced tropical cyclogenesis within an axisymmetric atmospheric balanced model. *Q. J. R. Meteorol. Soc., 132*, 2603–23.

Fraedrich, K. and McBride, J.L. 1989. Large-scale convective instability revisited. *J. Atmos. Sci., 52*, 1914–23.

Gentry, R.C., Fujita, T.T. and Sheets, R.C. 1970. Aircraft, spacecraft, satellite and radar observations of hurricane *Gladys*, 1968. *J. Appl. Meteor., 9*, 837–50.

Gopalakrishnan, S.G., Marks, F. JR., Zhang X., Bao J.W., Yeh, K.S., and Atlas R. 2011. The experimental HWRF system: a study on the influence of horizontal resolution on the structure and intensity changes in tropical cyclones using an idealised framework. *Mon. Wea. Rev., 139*, 1762–84.

Gopalakrishnan, G., Marks, F. JR., Zhang, J.A., Zhang, X., Bao, J. and Tallapragada, V. 2012. A study of the impacts of vertical diffusion on the structure and intensity of the tropical cyclones using the high resolution HWRF system. *J. Atmos. Sci.*, 70, 524–41.

Greenspan, H.P. and Howard, L.N. 1963. On a time-dependent motion of a rotating fluid. *J. Fluid Mech., 17*, 385–404.

Gray, S.L. and Craig G.C. 1998. A simple theoretical model for the intensification of tropical cyclones and polar lows. *Q. J. R. Meteorol. Soc., 124*, 919–47.

Guinn, T. and Schubert, W.H. 1993: Hurricane spiral bands. *J. Atmos. Sci., 50*, 3380–03.

Hendricks, E.A., Montgomery, M.T. and Davis, C.A. 2004. On the role of 'vortical' hot towers in formation of tropical cyclone *Diana* (1984). *J. Atmos. Sci., 61*, 1209–32.

Hill, K.A., Lackmann, G.M. 2009. Analysis of idealised tropical cyclone simulations using the Weather Research and Forecasting model: sensitivity to turbulence parameterisation and grid spacing. *Mon. Wea. Rev., 137*, 745–65.

Holton, J.R. 1992. *An introduction to dynamic meteorology*. Third edition. Academic Press, London, 507pp.

Holton J.R. 2004: *An introduction to dynamic meteorology*. Fourth edition. Academic Press, London, 535pp.

Holton J.R. and Hakim, G. 2012: *An introduction to dynamic meteorology* Fifth edition. Academic Press, London, 552pp.

Houze, R.A. jr. 2010 Clouds in tropical cyclones. *Mon. Wea. Rev., 138*, 293–344.

Houze, R.A., Lee W-C. and Bell M.M. 2009. Convective contribution to the genesis of hurricane *Ophelia* (2005). *Mon. Wea. Rev., 137*, 2778–00.

Howells, P.A.C., Rotunno, R. and Smith R.K. 1988. A comparative study of atmospheric and laboratory-analogue tornado-vortex models. *Q. J. R. Meteorol. Soc., 114*, 801–22.

James, I.N. 1994. *Introduction to circulating atmospheres*, Cambridge University Press, London, 422pp.

James, R.P. and Markowski, P.M. 2010. A numerical investigation of the effects of dry air aloft on deep convection. *Mon. Wea. Rev., 138*, 140–61.

Kepert, J.D. 2006a. Observed boundary-layer wind structure and balance in the hurricane core. Part I. Hurricane *Georges. J. Atmos. Sci., 63*, 2169–93

Kepert, J.D. 2006b. Observed boundary-layer wind structure and balance in the hurricane core. Part II. Hurricane *Mitch. J. Atmos. Sci., 63*, 2194–11

Kepert, J.D. 2011. Tropical cyclone structure and dynamics, *Global perspectives on Tropical cyclones: from science to mitigation* (eds. Chan and Kepert), World Scientific Series on Asia-Pacific Weather and Climate, Vol. 4, 448pp

Kepert, J.D. and Wang, Y. 2001. The dynamics of boundary layer jets within the tropical cyclone core. Part II: Nonlinear enhancement. *J. Atmos. Sci., 58*, 2485–01.

Kilroy, G. and Smith R.K. 2013. A numerical study of rotating convection during tropical cyclogenesis. *Q. J. R. Meteorol. Soc., 139*, 1255–69.

Kleinschmidt, K, 1951: Grundlagen einer Theorie der tropischen Zyklonen. *Arch. Meteor. Geophys. Bioklim., A4*, 53–72.

Kossin, J., McNoldy, B.D. and Schubert W.H. 2002: Vortical swirls in hurricane eye clouds. *Mon. Wea. Rev., 130*, 3144–49.

Kurihara, Y. and Tuleya R.E. 1974. Structure of a tropical cyclone developed in a three-dimensional numerical simulation model. *J. Atmos. Sci., 31*, 893–19.

Lewellen, D.C. and Lewellen, W.S. 2007a: Near-surface intensification of tornado vortices. *J. Atmos. Sci., 64*, 2176–94.

Lewellen, D.C. and Lewellen W S 2007b. Near-surface vortex intensification through corner flow collapse. *J. Atmos. Sci., 64*, 2195–09.

Lighthill J. 1998. Fluid mechanics of tropical cyclones. *Theoret. Comput. Fluid. Dyn., 10*, 3–21.

Malkus J.S. 1958. On the structure and maintenance of the mature hurricane eye. *J. Meteor., 15*, 337–49.

Malkus J.S., Riehl H. 1960. On the dynamics and energy transformations in steady-state hurricanes. *Tellus, 12*, 1–20.

Möller, J.D. and Smith R.K. 1994: The development of potential vorticity in a hurricane-like vortex. *Q. J. R. Meteorol. Soc., 120*, 1255–65.

Möller, J.D. and Montgomery M.T. 2000. Tropical cyclone evolution via potential vorticity anomalies in a three-dimensional balance model. *J. Atmos. Sci., 57*, 3366–87.

Möller, J.D. and Shapiro, L.J. 2002. Balanced contributions to the intensification of hurricane *Opal* as diagnosed from a GFDL model forecast. *Mon. Wea. Rev., 130*, 1866–81.

Marks, F.D. and Shay L.K. 1998. Landfalling tropical cyclones: forecast problems and associated research opportunities. *Bull. Amer. Meteor. Soc., 79*, 305–23.

Marks, F., Houze, R.A. and Gamache, J.F. 1992. Dual-aircraft investigation of the inner core of hurricane *Norbert*. Part I: Kinematic structure. *J. Atmos. Sci., 49*, 919–42.

Marks, F.D., Black P.G. Montgomery, M.T. and Burpee, R.W. 2008. Structure of the eye and eyewall of hurricane *Hugo* (1989). *Mon. Wea. Rev., 136*, 1237–59.

Marin, J.C., Raymond, D.J. and Raga, G.B. 2009. Intensification of tropical cyclones in the GFS model. *Atmos. Chem. Phys. Discuss., 9*, 1407–17.

Martinez, Y.H. 2008 *Diagnostic study of hurricane asymmetries using empirical normal modes*. Ph.D. thesis, McGill University, 165 pp.

Martinez, Y.H., Brunet, G., Yau, M.K. and Wang X. 2011. On the dynamics of concentric eyewall genesis: spacetime empirical normal modes diagnosis. *J. Atmos. Sci., 68*, 458–76.

Molinari, J.M., Skubis, S. and Vollaro, D. 1995: External influences on hurricane intensity. Part III: Potential vorticity structure. *J. Atmos. Sci., , 52*, 3593–06.

Molinari, J., Moore, P. and Idone V. 1999. Convective structure of hurricanes as revealed by lightning locations. *Mon. Wea. Rev., 127*, 520–34.

Molinari, J., Vollaro, and D. Corbosiero K.L. 2004. Tropical cyclone formation in a sheared environment: a case study. *J. Atmos. Sci., 61*, 2493–509.

Montgomery, M.T. and Kallenbach R.J. 1997. A theory for vortex Rossby waves and its application to spiral bands and intensity changes in hurricanes. *Q. J. R. Meteorol. Soc., 123*, 435–65.

Montgomery, M.T., Vladimirov, V.A. and Denissenko P.V. 2002. An experimental study on hurricane mesovortices. *J. Fluid. Mech., 471*, 1–32.

Montgomery, M.T., Nicholls, M.E., Cram, T.A. and Saunders, A.B. 2006: A vortical hot tower route to tropical cyclogenesis. *J. Atmos. Sci., 63*, 355–86.

Montgomery, M.T., Nguyen, S.V., Smith, R.K. and Persing, J. 2009. Do tropical cyclones intensify by WISHE? *Q. J. R. Meteorol. Soc., 135*, 1697–14.

Nguyen, C.M., Smith, R.K. Zhu, H. and Ulrich, W. 2002. A minimal axisymmetric hurricane model. *Q. J. R. Meteorol. Soc., 128*, 2641–61.

Nguyen, S.V., Smith, R.K., Montgomery, M.T. 2008. Tropical cyclone intensification and predictability in three dimensions. *Q. J. R. Meteorol. Soc., 134*, 563–82.

Nguyen, C.M., Reeder, M.J., Davidson, N.E., Smith, R.K. and Montgomery M.T. 2011. Inner-core vacillation cycles during the intensification of hurricane *Katrina*. *Q. J. R. Meteorol. Soc., 137*, 829–44.

Nolan, D.S. and Montgomery, M.T. 2000. The algebraic growth of wavenumber one disturbances in hurricane-like vortices, *J. Atmos. Sci., 57*, 3514–38.

Nolan, D.S., Montgomery, M.T. and Grasso, L.D. 2001. The wavenumber-one instability and trochoidal motion of hurricane-like vortices. *J. Atmos. Sci., 58*, 3243–70.

Nolan, D.S., Zhang, J.A. and Stern, D.P. 2009a. Evaluation of planetary boundary layer parameterisations in tropical cyclones by comparison of in-situ observations and high-resolution simulations of hurricane *Isabel* (2003). Part I: Initialization, maximum winds, and the outer core boundary layer. *Mon. Wea. Rev., 137*, 3651–74.

Nolan, D.S., Zhang, J.A. and Stern, D.P. 2009b. Evaluation of planetary boundary layer parameterizations in tropical cyclones by comparison of in-situ observations and high-resolution simulations of hurricane *Isabel* (2003). Part II: Inner core boundary layer and eyewall structure. *Mon. Wea. Rev., 137*, 3675–98.

Nong, S. and Emanuel, K.A. 2004. A numerical study of the genesis of concentric eyewalls in hurricanes. *Q. J. R. Meteorol. Soc., 129*, 3323–38.

Ooyama, K.V. 1964. A dynamical model for the study of tropical cyclone development. *Geophys. Int., 4*, 187–98.

Ooyama, K.V. 1969. Numerical simulation of the life cycle of tropical cyclones. *J. Atmos. Sci., 26*, 3–40.

Ooyama K.V. 1982. Conceptual evolution of the theory and modeling of the tropical cyclone. *J. Meteor. Soc. Japan, 60*, 369–80.

Ooyama, K.V., 1997. Footnotes to ‘Conceptual Evolution’. Extended Abstracts, *22nd Conference on Hurricanes and Tropical Meteorology*, American Meteorological Society, Boston, 13–18.

Persing, J. and Montgomery, M.T. 2003. Hurricane superintensity. *J. Atmos. Sci., 60*, 2349–71.

Persing, J., Montgomery, M.T., McWilliams, J. and Smith R.K. 2013.Asymmetric and axisymmetric dynamics of tropical cyclones. *Atmos. Chem. Phys. 13*, 12299–341.

Plougonven, R. and Zeitlin, V. 2002. Internal gravity wave emission from a pancake vortex: an example of wave vortex interaction in strongly stratified flows. *Phys. Fluids, 14*, 1259–68.

Powell, M.D., Uhlhorn, E.W., Kepert, J.D. 2009. Estimating maximum surface winds from hurricane reconnaissance measurements. *Wea. Forecasting, 24*, 868–83.

Rauber, R. M., Walsh, J.E., and Charlevoix, D.J. 2009. *Severe and hazardous weather: an introduction to high impact meteorology*. Third edition. Kendall/Hunt Publishing Company, 642 pp.

Raymond, D.J., Emanuel, K.A. 1993. The Kuo cumulus parameterisation, *The representation of cumulus convection in numerical models*. Meteorological Monograph No. 46, American Meteorological Society, Boston, Mass. USA, pp145–47.

Raymond, D.J., López-Carrillo, C. and Cavazos, L.L. 1998, Case studies of developing east Pacific easterly waves. *Q. J. R. Meteorol. Soc., 124*, 2005–34.

Raymond, D.J., Sessions, S.L. and Fuchs, Z. 2007, A theory for the spin-up of tropical depressions. *Q. J. R. Meteorol. Soc., 133*, 1743–54.

Raymond, D.J. and López Carillo, C. 2011. The vorticity budget of developing typhoon *Nuri* (2008). *Atmos. Chem. Phys., 11*, 147–63.

Reasor, P.D. and Montgomery, M.T. 2001. Three-dimensional alignment and co-rotation of weak TC-like vortices via linear vortex Rossby waves. *J. Atmos. Sci., 58*, 2306–30.

Reasor, P.D., Montgomery, M.T. and Bosart L. 2005. Mesoscale observations of the genesis of hurricane *Dolly* (1996). *J. Atmos. Sci., 62*, 3151–71.

Riehl, H. 1954. *Tropical meteorology*. McGraw-Hill, 392pp.

Riemer, M., Montgomery, M.T. and Nicholls, M.E. 2010. A new paradigm for intensity modification of tropical cyclones: thermodynamic impact of vertical wind shear on the inflow layer, *Atmos. Chem. and Phys., 10*, 3163–88.

Rosenthal, S.L. and Koss, W.J. 1968. Linear analysis of a tropical cyclone model with increased vertical resolution. *Mon. Wea. Rev., 96*, 858–966.

Rotunno, R. and Emanuel, K.A. 1987. An air-sea interaction theory for tropical cyclones. Part II: Evolutionary study using a nonhydrostatic axisymmetric numerical model. *J. Atmos. Sci., 44*, 542–61.

Rotunno, R., Chen, W., Wang, W. Davis, C. Dudhia, J. and Holland G.H. 2009. Large-eddy simulation of an idealised tropical cyclone. *Bull. Amer. Meteor. Soc., 90*, 1783–88.

Sanger, N.T., Montgomery, M.T., Smith, R.K. and Bell, M.M. 2014: An observational study of tropical cyclone spin up in supertyphoon Jangmi (2008) from 24 to 27 September. *Mon. Wea. Rev., 142*, 3–28.

Schecter, D.A. and Montgomery, M.T. 2007. Waves in a cloudy vortex. J. *Atmos. Sci., 64*, 314–37.

Schubert, W.H. and Alworth, B.T. 1987 Evolution of potential vorticity in tropical cyclones. *Q. J. R. Meteorol. Soc., 113*, 147–62.

Schubert, W.H., Montgomery, M.T., Taft, R.K., Guinn, T.A. Fulton, S.R. Kossin, J.P. and Edwards J.P. 1999 Polygonal eyewalls, asymmetric eye contraction, and potential vorticity mixing in hurricanes. *J. Atmos. Sci., 56*, 1197–23.

Schubert, W.H., Rozoff, C.M., Vigh, J.L., McNoldy, B.D. and Kossin J.P. 2007 On the distribution of subsidence in the hurricane eye. *Q.J. R. Meteorol. Soc.*, 133, 1–20.

Schwendike, J. and Kepert, J.D. 2008. The boundary layer winds in hurricane *Danielle* (1998) and *Isabel* (2003). *Mon. Wea. Rev., 136*, 3168–92.

Shapiro, L.J. 1983. The asymmetric boundary layer flow under a translating hurricane. *J. Atmos. Sci., 40*, 1984–98.

Shapiro, L.J. and Willoughby, H. 1982. The response of balanced hurricanes to local sources of heat and momentum. *J. Atmos. Sci., 39*, 378–94.

Shapiro, L.J. and Montgomery, M.T. 1993. A three-dimensional balance theory for rapidly-rotating vortices. *J. Atmos. Sci., 50*, 3322–35.

Shin, S. and Smith, R.K. 2008 Tropical-cyclone intensification and predictability in a minimal three-dimensional model, *Q. J. R. Meteorol. Soc., 134*, 1661–71.

Sippel, J.A., Nielsen-Gammon, J.W. and Allen, S.E. 2006: The multiple vortex nature of tropical cyclogenesis. *Mon. Wea. Rev., 134*, 1796–14.

Skamarock, W.C., Klemp, J.B., Dudhia, J., Gill, D.O., Barker, D.M., Wang, W. and Powers, J.G. 2005. *A description of the Advanced Research WRF: Version 2*. NCAR Tech. Note 468 1 STR, 88pp.

Smith, R.K. and Leslie, L.M. 1976. Thermally driven vortices: a numerical study with application to dust devil dynamics. *Q. J. R. Meteorol. Soc., 102*, 791–04.

Smith, R.K. 1980. Tropical cyclone eye dynamics, *J. Atmos. Sci., 37*, 1227–32.

Smith, R.K. 1997. On the theory of CISK. *Q. J. R. Meteorol. Soc., 123*, 407–18.

Smith, R.K. 2000. The role of cumulus convection in hurricanes and its representation in hurricane models. *Rev. Geophys., 38*, 465–89.

Smith, R.K. 2003. A simple model of the hurricane boundary layer. *Q. J. R. Meteorol. Soc., 129*, 1007–27.

Smith, R.K. 2006. Accurate determination of a balanced axisymmetric vortex. *Tellus, 58A*, 98–103.

Smith, R.K., 2007. A balanced axisymmetric vortex in a compressible atmosphere: corrigendum and addendum. *Tellus, 59A*, 785–86.

Smith, R.K. and Ulrich, W. 1990. An analytical theory of tropical cyclone motion using a barotropic model. *J. Atmos. Sci., 47*, 1973–86.

Smith, R.K. and Ulrich, W. 1993. Vortex motion in relation to the absolute vorticity gradient of the environment. *Q. J. R. Meteorol. Soc., 119*, 207–15.

Smith, R.K. and Vogl, S. 2008. A simple model of the hurricane boundary layer revisited. *Q. J. R. Meteorol. Soc., 134*, 337–51.

Smith, R.K. and Montgomery, M.T. 2008. Balanced boundary layers in hurricane models. *Q. J. R. Meteorol. Soc., 134*, 1385–95.

Smith, R.K. and Montgomery, M.T. 2010. Hurricane boundary- layer theory. *Q. J. R. Meteorol. Soc., 136*, 1665–70.

Smith, R.K. and Thomsen G.L. 2010. Dependence of tropical-cyclone intensification on the boundary-layer representation in a numerical model. *Q. J. R. Meteorol. Soc., 136*, 1671–85.

Smith, R.K. and Montgomery, M.T. 2013.On the existence of the logarithmic surface layer in hurricanes. *Q. J. R. Meteorol. Soc., 140*, 72–81

Smith, R.K., Ulrich, W. and Dietachmayer, G. 1990. A numerical study of tropical cyclone motion using a barotropic model. PartI. The role of vortex asymmetries. *Q. J. R. Meteorol. Soc., 116*, 337–62.

Smith, R.K., Montgomery, M.T. and Zhu, H. 2005: Buoyancy in tropical cyclones and other rapidly rotating vortices. *Dyn. Atmos. Oceans, 40*, 189–208.

Smith, R.K., Montgomery, M.T. and Vogl, S. 2008. A critique of Emanuel's hurricane model and potential intensity theory. *Q. J. R. Meteorol. Soc., 134*, 551–61.

Smith, R.K., Montgomery, M.T. and Nguyen, S.V. 2009: Tropical-cyclone spin up revisited. *Q. J. R. Meteorol. Soc., 135*, 1321–35.

Sundqvist, H. 1970. Numerical simulation of the development of tropical cyclones with a ten-level model. Part I. *Tellus, 22*, 359–89.

Tang, B. and Emanuel, K.A. 2010. Mid-level ventilation's constraint on tropical-cyclone intensity. *J. Atmos. Sci., 67*, 1817–30.

Terwey, W.D. and Montgomery, M.T. 2008. Secondary eyewall formation in two idealised, full-physics modeled hurricanes. *J. Geophys. Res., 113*, D12112 – 3.0.

Vogl, S. and Smith, R.K. 2009. A simple model of the hurricane boundary layer revisited. *Q. J. R. Meteorol. Soc., 135*, 337–51.

Wang, Y. and Holland, G.J. 1996a. The beta drift of baroclinic vortices. Part I: Adiabatic vortices. *J. Atmos. Sci., 53*, 411–27.

Wang, Y. and Holland, G.J. 1996b. The beta drift of baroclinic vortices. Part II: Diabatic vortices. *J. Atmos. Sci., 53*, 3737–56.

Wang, Y. 2001. An explicit simulation of tropical cyclones with a triply nested movable mesh primitive equation model: TCM3, Part I: Model description and control experiment. *Mon. Wea. Rev., 129*, 1370–94.

Wang, Y. 2002a: Vortex Rossby waves in a numerical simulated tropical cyclone. Part I: Overall structure, potential vorticity, and kinetic energy budgets. *J. Atmos. Sci., 59*, 1213–38.

Wang, Y. 2002b. Vortex Rossby waves in a numerical simulated tropical cyclone. Part II: The role in tropical cyclone structure and intensity changes. *J. Atmos. Sci., 59*, 1239–62.

Wang, Y. and Wu, C-C. 2004. Current understanding of tropical cyclone structure and intensity changes—a review. *Meteorol. Atmos. Phys., 87*, 257–78.

Wang, Y. 2008. Rapid filamentation zone in a numerically simulated tropical cyclone model. *J. Atmos. Sci., 65*, 1158–81.

Weckwerth, T.M. 2000. The effect of small-scale moisture variability on thunderstorm initiation. *Mon. Wea. Rev., 128*, 4017–30.

Willoughby, H.E. 1979: Forced secondary circulations in hurricanes. *J. Geophys. Res.*, 84, 3173–83.

Willoughby, H.E. 1988: The dynamics of the tropical cyclone core. *Aust. Met. Mag., 36*, 183–91.

Willoughby, H.E. 1990a. Gradient balance in tropical cyclones. *J. Atmos. Sci., 47*, 465–89.

Willoughby, H.E. 1990b. Temporal changes of the primary circulation in tropical cyclones. *J. Atmos. Sci., 47*, 242–64.

Willoughby, H.E. 1995. Mature structure and evolution, *Global perspectives on propical cyclones*, pp. 21-62. WMO/TD- No 693 (Ed. R. L. Elsberry), World Meteorological Organization, Geneva, 289pp.

Wissmeier, U., Smith, R.K., 2011: Tropical-cyclone convection: the effects of ambient vertical vorticity. *Q. J. R. Meteorol. Soc., 137*, 845–57.

Yang, B., Wang, Y. and Wang, B. 2007.The effects of internally generated asymmetries on tropical cyclone potential intensity. *J. Atmos. Sci.*, *64*, 1165–88.

Yano, J-I. and Emanuel, K.A. 1991. An improved model of the equatorial troposphere and its coupling with the stratosphere. *J. Atmos. Sci.*, *48*, 377-389.

Zhang, D-L., Liu Y. and Yau, M.K. 2001. A multi-scale numerical study of hurricane *Andrew* (1992). Part IV: Unbalanced flows. *Mon. Wea. Rev.*, *129*, 92–107.

Zhang, J., Drennan, W.M., Black, P.B. and French, J.R. 2009 Turbulence structure of the hurricane boundary layer between the outer rainbands. *J. Atmos. Sci.*, *66*, 2455–67.

Zhang, J.A., Rogers, R.F., Nolan, D.S. and Marks, F.D. 2011a. On the characteristic height scales of the hurricane boundary layer. *Mon. Wea. Rev.*, *139*, 2523–35.

Zhang, J.A., Marks, F.D., Montgomery, M.T. and Lorsolo, S. 2011b. An estimation of turbulent characteristics in the low-level region of intense hurricanes *Allen* (1980) and *Hugo* (1989). *Mon. Wea. Rev.*, *139*, 1447–62.

Zhang, J.A. and Drennan, W.A. 2012. An observational study of vertical eddy diffusivity in the hurricane boundary layer. J. *Atmos. Sci.*, *69*, 3223–36.

Zhang, J.A. and Montgomery, M.T. 2012. Observational estimates of the horizontal eddy diffusivity and mixing length in the low-level region of intense hurricanes. *J. Atmos. Sci.*, *69*, 1306–16.

Zhu, H. and Smith, R.K. 2002. The importance of three physical processes in a three-dimensional tropical cyclone model. *J. Atmos. Sci.*, *59*, 1825–40.

Zhu, H., and Smith, R.K. 2003. Effects of vertical differencing in a minimal hurricane model. *Q. J. R. Meteorol. Soc.*, *129*, 1051–69.

Zhu, H., Smith, R.K. and Ulrich W. 2001. A minimal three-dimensional tropical cyclone model. *J. Atmos. Sci.*, *58*, 1924–44.